Pooling Cherries and Lemons

Some simple economics of complex financial products*

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Abstract

We study banks' incentives to create complex securities backed by assets of heterogeneous quality. When banks offer complex securities, investors disagree on their value. Disagreement is beneficial to the banks when investors are wealthy enough since then prices are determined by more optimistic investors. In bad times instead, banks prefer to offer simple securities as these create no disagreement. The incentives to issue complex securities are exacerbated in more competitive banking environments. Moreover, competition may induce a form of prisoners' dilemma in which banks end up issuing complex securities while they would be better off if they all committed to issuing simple securities.

Keywords: securitization, complexity, disagreement, bounded rationality, market efficiency.

JEL codes: C72; D53; G14; G21

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1 Introduction

Investors daily trade structured financial products for hundreds of billions of dollars.[1] Financial institutions create such products by pooling multiple financial assets and by designing securities whose payoffs depend on the value of the underlying assets. Such a practice, called securitization, has a long history and it has been traditionally viewed as an instrument to create liquidity and share risk.[2] Over the recent past, structured financial products have taken forms of increasing complexity (Zandi (2008)). Notable commentators have argued that this complexity has led to an increased exposure to risk for investors, who may end up buying assets they do not understand.[3]

The complexity of these products makes it challenging for investors to define the right valuation model. One implication is that even starting with the same objective information, investors may end up with very different assessments.[4] A number of studies have documented a considerable amount of heterogeneity in the evaluations of asset-backed securities even among highly sophisticated investors. These studies suggest that the heterogeneity is most likely driven by the use of different valuation methods (Bernardo and Cornell (1997); Carlin, Longstaff and Matoba (2014)).

Another implication is that issuers may have the incentive to tailor the complexity of their securities so as to take advantage of the imperfect models employed by investors. In his popular account of the 2008 crisis, Michael Lewis (2010) suggests that rating agencies were not paying attention to every loan in a given pool but rather focused on a "representative asset" in the pool. In response, issuers created pools with assets of very heterogeneous quality, which resulted in larger

[1]In the U.S. in 2015, the average daily trading volume of securitized products was \$196.7 billion. That makes it the second largest fixed-income market after the Treasury bond market (see SIFMA's *2015 Securitization Year in Review* at www.sifma.org).

[2]Fratianni (2006) describes securitization in the 12th century in Genoa. Frehen, Goetzmann and Rouwenhorst (2013), Riddiough and Thompson (2012), Choudry and Buchanan (2014) discuss other famous historical examples.

[3]Paul Krugman (2007): *"[Asset backed securities] were promoted as ways to spread risk, making investment safer. What they did instead -aside from making their creators a lot of money [..]- was to spread confusion, luring investors into taking on more risk than they realized."* George Soros (2009): *"Securitization was meant to reduce risks [..]. As it turned out, they increased the risks by transferring ownership of mortgages from bankers who knew their customers to investors who did not."*

[4]Mark Adelson (S&P chief credit officer): *"It [Complexity] is above the level at which the creation of the methodology can rely solely on mathematical manipulations. Despite the outward simplicity of credit-ratings, the inherent complexity of credit risk in many securitizations means that reasonable professionals starting with the same facts can reasonably reach different conclusions."* Testimony before the Committee on Financial Services, U.S. House of Representatives, September 27, 2007. Quoted in Skreta and Veldkamp (2009).

profits for the issuers and larger risks for investors.[5]

These observations have led us to the following research questions. If professionals or investors evaluate complex securities using simplified models, what is the effect on asset prices? In turn, how does this affect banks' strategies of designing more or less complex securities? Are banks' strategies likely to be different in good and bad times? Are the incentives to create complex securities reduced or exacerbated by competition among banks?

We develop a simple model to address these questions. We consider several banks holding assets (say, loan contracts) of different quality (say, probability of default). Banks are able to package their assets into pools as they wish and sell securities backed by these pools. Securities can be made more or less complex, and in our setting (as it will become clear) the complexity of a given security depends on the heterogeneity of the assets in the underlying pool. The familiar market clearing conditions determine the prices of the various securities assuming investors are wealth-constrained and cannot short-sell (an assumption we relax in the extension section).

Our key assumption is that investors have a limited ability to assess the value of the pools, which we describe as follows: Each investor randomly samples one asset from each pool and assumes that the average value of the assets in the pool coincides with this draw considered as "representative." If an investor samples an asset with expected value $\tilde{x}$ from pool α, this investor believes that on average assets in α have expected value $\tilde{x}$. No other information is used for assessing the value of pool α. In particular, investors do not consider how banks may strategically allocate assets into pools, nor do they draw any inference from market prices.

This heuristic describes in a simple (and stylized) way a form of extrapolation from the limited amount of information investors are able to process. Our investors excessively focus on their sampled signal and do not take into account other relevant information. Most notably, investors overlook banks' incentives to tailor the compositions of the pools, and this is key to determine how "representa-

[5] Lewis (2010): *"All they [Moody's and S&P] and their models saw, and evaluated, were the general characteristics of loan pools. To meet the rating agencies' standards, [..]the average FICO score of the borrowers in the pool needed to be around 615. There was more than one way to arrive at that average number. And therein lay a huge opportunity. A pool of loans composed of borrowers all of whom had a FICO score of 615 was far less likely to suffer huge losses than a pool of loans composed of borrowers half of whom had FICO scores of 550 and half of whom had FICO scores of 680. [...] Barbell-shaped loan pools, with lots of very low and very high FICO scores in them, were a bargain [for banks] compared to pools clustered around the 615 average."*

tive" an asset can be relative to the underlying pool.[6] This is precisely what makes investors' heuristic vulnerable to manipulation. Further discussion about how the heuristic departs from the standard rationality framework appears in Section 2, after the presentation of the model.[7]

We further assume that the draws determining the representative samples are made independently across investors. This is a simple way to capture that as the underlying assets become more heterogeneous -or, equivalently, as securities become more complex- their evaluations are more dispersed across investors, in line with the above mentioned evidence.[8] By contrast, if assets are sold separately -or in homogeneous packages- the same sampling procedure leads investors to the correct evaluation, and there is no disagreement. Thus, disagreement among investors is endogenously shaped by banks' securitization strategies.

Disagreement may or may not be beneficial to the banks. Some investors tend to overestimate and other investors tend to underestimate the value of the pool (across investors, estimations are on average correct). The key question for banks is whether market clearing prices are determined by more or less optimistic investors. In our model, the answer depends on investors' wealth. We assume that each investor is risk neutral and so allocates his whole wealth to the securities perceived as most underpriced. When investors are wealthy, only a small fraction of them is needed to buy the securities at high prices. The larger the wealth, the more optimistic the marginal investor who determines the market clearing price, which in turns increases the incentive for banks to create disagreement by selling complex securities.

We first study the case of a monopolistic bank. In the extreme case in which investors are very wealthy, only the most optimistic investors buy, and their wealth is so large that the bank wishes to create as much disagreement as possible. This is achieved by selling securities backed by the most heterogeneous pool, the one which contains all assets in the bank. More generally, we show that the larger the aggregate wealth, the more profitable it is for the bank to create more complex securities. Conversely, when wealth falls below some threshold, the bank prefers

[6] If banks were to pool homogenous assets, one draw would be highly representative of the assets in the pool. If banks instead tend to pool assets of heterogeneous quality (as we show they do) this is no longer the case.

[7] Our model assumes on the other hand that banks are perfectly rational. This does not necessarily require that banks are fully aware also of the valuation methods used by investors. One can view banks' strategies as the result of a (possibly long) trial and error process in which banks would have eventually learned the best securitization method.

[8] More generally, the insights developed below would carry over, as long as there is no perfect correlation of the draws across investors.

to not securitize, and it rather sells the loans as separate assets. We also provide further insights on the optimal composition of the bundles: we show that it is typically not profitable for the bank to pool assets of homogenous quality and that bundles tend to contain worst quality assets.

A second key determinant of banks' strategies is the degree of market competition. We show that, when several banks compete to attract investors' wealth, the incentives to create complex securities are increased. To gain some intuition behind the result, suppose that each bank pools all its assets and, given investors' wealth, suppose the resulting securities are sold at a price equal to the fundamental, i.e. the average value of the assets in the pool. If one bank deviates and sells its assets separately, it would receive a lower price for its low quality assets. At the same time, it would not be able to sell its high quality assets at a higher price. This is because the market price of the separate assets of high quality would be determined by the many investors who sample one good asset from at least one pool (the more banks, the more such investors), thereby making the high quality asset of the deviating bank indistinguishable from the pools of the other banks.[9] This in turn would make the deviation non-profitable, and all banks would go for the securitization strategy.

More generally, we show that the strategic interaction among banks decreases the lower bound on aggregate wealth above which banks prefer to securitize. This suggests that if one wishes to regulate the use of securitization, competition alone does not seem to be of much use. Instead, competition induces a form of persistence in the practice as there are levels of wealth at which oligopolistic banks are induced to create complex securities while a monopolist would sell the assets separately.

We also show that securitization creates a negative externality on the other banks. When a bank chooses to offer complex securities, it reduces the amount of wealth investors can use to buy the remaining assets and that is detrimental to the other banks. Irrespective of its strategy, a bank is better off if the other banks do not securitize and sell their assets separately.

Due to this externality, banks may face a kind of prisoner's dilemma in which the only equilibrium requires banks to offer complex securities, and that is detrimental to the banks in the sense that banks would sell at higher prices if they could jointly decide to not securitize. When banks have the option to withhold

[9] If all these investors preferred the pools, the corresponding securities would become so expensive that it would be profitable to buy the separate asset instead, and conversely if they all preferred the separate asset. Market clearing requires these investors to be indifferent.

their assets (possibly at a cost), such low prices may lead banks to limit what they put on the market, thereby resulting in welfare losses.

Empirical Implications

The previous insights provide a rationale for several empirical observations as well as novel predictions on the practice of securitization and on the pricing of the corresponding securities. We suggest why securitization tends to develop in good times (banks take advantage of securitization when investors have a lot of capital to invest) and to drop when wealth in the economy shrinks. Such boom and bust have been observed quite clearly in relation to the 2008 crisis (Gorton and Metrick (2012)). Our framework can serve as a building block for more systematic investigations of the incentives for securitization along the business cycle.

We suggest why banks have incentives to bundle assets of heterogeneous quality, as the barbell-shaped pools in the above mentioned quote by Lewis, and why low quality assets tend to be sold in bundles (for evidence of this, see e.g. Downing, Jaffee and Wallace (2009), Gorton and Metrick (2012), Ghent, Torous and Valkanov (2014)). We also predict that the heterogeneity of the pools tends to be larger in good times, which we believe has not yet been tested.

In terms of pricing, a key feature of our model is that prices tend to be high when a few investors buy the assets, which generates a negative relation between breadth of ownership and asset returns. Such a relation has been documented for stocks in Chen, Hong and Stein (2002) who also consider the roles of heterogeneous beliefs and short sale constraints. We are not aware of similar tests on asset-backed securities.

We also predict a positive relation between asset complexity and overpricing. The relation has been documented in several empirical studies, suggesting that issuers deliberately create complex securities which investors fail to fully understand (see Lewis (2010) but also Henderson and Pearson (2011), Furfine (2014), Ghent et al. (2014) and Célérier and Vallée (2015)). Specifically, our analysis provides a precise link between asset complexity, investors' disagreement and asset prices which should be subject of future tests.

Literature on Extrapolation, Misvaluation and Disagreement

In the modelling of investors' heuristics, we build on a large literature documenting that many investors tend to extrapolate from the limited amount of information they hold. Our modelling is in line with the representativeness heuristic discussed in the psychology literature (see Tversky and Kahneman (1975)). In particular, it describes what Tversky and Kahneman (1971) called the "law of small numbers" whereby "people regard a sample randomly drawn from a population as highly representative, that is, similar to the population in all essential characteristics." In financial markets, extrapolation has been documented in surveys on investors' expectations (Shiller (2000); Dominitz and Manski (2011); Greenwood and Shleifer (2014)) as well as in actual investment decisions (Benartzi (2001); Greenwood and Nagel (2009); Baquero and Verbeek (2008)). Our formalization is most similar to Spiegler (2006) and Bianchi and Jehiel (2015), but several other models of extrapolative investors have been developed including De Long, Shleifer, Summers and Waldmann (1990); Barberis, Shleifer and Vishny (1998); Rabin (2002) and Rabin and Vayanos (2010). An important distinctive feature of our model is that, as mentioned, how "representative" a signal is relative to the underlying population is a function of banks' strategies. In addition, none of these models addresses securitization.

In recent years, asset-backed securities have been evaluated also through specialized agents such as financial advisors or rating agencies. As mentioned above, while their models are certainly more sophisticated, they were not well suited to deal with the typical complexity of these securities.[10] A common theme in the 2008 crisis was that models in place were appropriate to evaluate much simpler securities (like corporate bonds) and, when applied to asset-backed securities, they overlooked fundamental specificities of these securities and led investors to misperceive risk (Mason and Rosner (2007), Coval, Jurek and Stafford (2009), Gennaioli, Shleifer and Vishny (2012)).[11] Evidence suggests that inaccuracies were more pronounced for more complex assets (Opp, Opp and Harris (2013), Efing and Hau (2015)). Despite focusing on different dimensions of misperception (we abstract from misperception of risk in our baseline model by considering risk neutral investors), our assumed heuristic is in line with this narrative: our investors hold a model which works well when evaluating simple assets (i.e., single loans) but turns out to be inaccurate when applied to complex securities. This also connects our study to models of mispricing with rating agencies (as Skreta and Veldkamp (2009) and Bolton, Freixas and Shapiro (2012)). The mechanisms we propose are however very different, in particular, in that the complexity of asset-backed securities is endogenously chosen by the banks in our setting.

[10]As shown in Rajan, Seru and Vig (2015), banks strategic responses to investors' models also contribute to the inherent complexity of these instruments, making the task of defining the correct valuation model even more challenging.

[11]See also Broer and Kero (2016) for a model in which asset backed securities are mispriced since investors (exogenously) disagree about risk.

Our model is also related to the literature on financial markets with heterogeneous beliefs and short selling constraints as in Harrison and Kreps (1978). Xiong (2013) provides a recent review and Simsek (2013) studies more specifically financial innovation in such markets. Methodologically, our approach differs from that literature in two important ways. First, the heterogeneity of beliefs in our setting is not a primitive of the model but it is endogenously determined by the bundling decision of banks. Second, the prices of assets in our model are not necessarily determined by the most optimistic investors, as the marginal investors fixing the level of prices depend on the level of wealth in the economy. Another interesting observation in relation to this literature is that introducing short selling in our model may *increase* the incentives to create disagreement in a market with several banks (see Section 6 for details).

Literature on Securitization and Bundling

A large literature on security design shows that an informed issuer may reduce adverse selection costs and promote trade by pooling its assets and create securities whose evaluations are less sensitive to private information. This typically takes the form of bundling all assets into one pool, slice the pool into a junior and a senior tranche, and sell the senior tranche to investors (see e.g. Myers and Majluf (1984), DeMarzo and Duffie (1999), Biais and Mariotti (2005), DeMarzo (2005)).[12]

As shown in Arora, Barak, Brunnermeier and Ge (2011), however, the argument is *reversed* when issuers can choose the content of the pools and create complex securities from these pools (as in our model): Asymmetric information is exacerbated and a complexity premium arises.[13] In our main analysis, we abstract from tranching possibilities. However, in Section 6, we show that tranching in our setting would allow banks to further exploit belief heterogeneity as opposed to providing better risk sharing.

Finally, the potential benefits of bundling have been studied in several other streams of literature, from IO to auctions. A common theme is that in face of uncertain valuations, some form of bundling is desirable because it allows the monopolist to reduce the informational rent left to the agents.[14] Our rationale is

[12] Recent contributions along these lines include Pagano and Volpin (2012), who show that selling opaque bundles is desirable when investors have different cognitive abilities as it reduces the informational advantage of the more sophisticated investors, and Farhi and Tirole (2015), who show that bundling induces the seller and the buyer to be symmetrically informed when both parties can acquire information.

[13] See also Sato (2014) for a model in which the complexity premium arises from investors' attempts to evaluate the quality of fund managers.

[14] In the context of a monopolist producing multiple goods, see e.g. Adams and Yellen (1976)

different, since in our model it is the mere choice of bundling that endogenously creates the dispersion of beliefs.

2 Model

There are N risk-neutral banks. Each bank i possesses several assets, we denote asset j of bank i as X_j^i and its expected payoff as x_j^i. For concreteness, asset X_j^i may be thought of as a loan contract with face value normalized to 1, probability of default $1 - x_j^i \in [0, 1]$, and zero payoff upon default. We order assets in terms of increasing expected payoff; that is, we have $x_j^i \leq x_{j+1}^i$ for each i and j.

Each bank may pool some of its assets and create securities backed by these pools. Each bank can package its assets into pools as it wishes. We denote by α_r^i a generic pool of bank i, which we identify as a subset of $X^i = \{X_j^i, j = 1, ..., J\}$. The bank then creates pass-through securities backed by the pool α_r^i: An investor who buys a fraction ω of the securities backed by α_r^i is entitled to a fraction ω of the payoffs generated by all the assets in α_r^i. Hence, the selling strategy of bank i can be represented simply as a partition of X^i which specifies which assets are put in the market and how those assets are bundled. We denote such a partition by $\alpha^i = \{\alpha_0^i, \{\alpha_r^i\}_r\}$, in which the set of assets which are kept in the bank is α_0^i and the set of bundles used for the corresponding securities are indexed by $r = 1, 2, ...$ The expected payoff of bank i choosing α^i is defined as

$$\pi^i = \sum_r \left|\alpha_r^i\right| p(\alpha_r^i) + \sum_{x_j^i \in \alpha_0^i} \tau x_j^i, \tag{1}$$

where $|\alpha_r^i|$ is the number of assets contained in α_r^i, $p(\alpha_r^i)$ is the price of the security backed by α_r^i and τx_j^i is the value of keeping asset X_j^i in the bank. In the case when $\tau = 0$, all assets have no value for the bank (say, an extreme form of liquidity shock). In the case when $\tau = 1$, the bank sells an asset only if the expected price is above the fundamental value. We denote the set of bundles sold by all banks as $A = \{\{\alpha_r^i\}_r\}_{i=1}^N$.

There are K risk-neutral investors, indexed by k. We denote by w_k the budget of investor k and by W the aggregate budget across all investors. That is, $W = \sum_k w_k$. Having in mind that the value of a bundle α_r^i may be difficult to assess, we assume that each investor uses the following heuristic procedure. For each bundle

and McAfee, McMillan and Whinston (1989). For models of auctions, see e.g. Palfrey (1983) and Jehiel, Meyer-Ter-Vehn and Moldovanu (2007).

α_r^i, investor k samples one basic asset from α_r^i at random (uniformly over all assets in α_r^i) and assumes that the average expected value of the assets in α_r^i coincides with this draw.[15] Specifically, denote by $\widetilde{x}_k(\alpha_r^i)$ the evaluation that investor k attaches to the average asset in α_r^i: $\widetilde{x}_k(\alpha_r^i)$ takes value x_j^i with probability $1/\left|\alpha_r^i\right|$ for every $X_j^i \in \alpha_r^i$. We assume that the draws are independent across investors. It follows that if $|\alpha_r^i| = 1$, investors share the same correct assessment of bundle α_r^i; if $|\alpha_r^i| > 1$, investors attach different values to α_r^i depending on their draw.

In what follows, we say that a security backed by α_r^i is simple when $|\alpha_r^i| = 1$, and that it is complex when $|\alpha_r^i| > 1$. Complexity considerations in our model arise merely from banks' strategies of how to bundle their assets, abstracting from elaborate tranching options which are briefly discussed in Section 6 (and shown to reinforce our arguments).

We allow the number of investors K to be finite (in which case one can think of dispersed investors following the advice of one financial expert out of K financial advisors) or to be infinite. When K is infinite, we have in mind the limiting case as $K \to \infty$ of a setting in which, from the law of large numbers, each asset in bundle α_r^i is sampled by a fraction $1/\left|\alpha_r^i\right|$ of investors.

Prices are determined by market clearing, where the supply and demand of the securities backed by α_r^i are defined as follows. If α_r^i consists of $|\alpha_r^i|$ assets, the supply of α_r^i is

$$S(\alpha_r^i) = \left|\alpha_r^i\right|. \tag{2}$$

The demand for α_r^i is defined as

$$D(\alpha_r^i) = \frac{1}{p(\alpha_r^i)} \sum_k w_k \lambda_k(\alpha_r^i), \tag{3}$$

where $\lambda_k(\alpha_r^i)$ is the fraction of the budget w_k allocated to bundle α_r^i. We assume no borrowing or short-selling (an assumption we discuss in Section 6), so that $\lambda_k \in [0,1]$ for all k. Given the risk-neutrality assumption, each investor allocates his entire budget to the securities perceived as most profitable. That is, we have

$$\lambda_k(\hat{\alpha}) > 0 \text{ iff } \hat{\alpha} \in \arg\max_{\alpha_r^i \in A} \frac{\widetilde{x}_k(\alpha_r^i)}{p(\alpha_r^i)} \text{ and } \widetilde{x}_k(\hat{\alpha}) - p(\hat{\alpha}) \geq 0,$$

and

$$\Sigma_{\alpha_r^i} \lambda_k(\alpha_r^i) = 1 \text{ if } \max_{\alpha_r^i \in A}(\widetilde{x}_k(\alpha_r^i) - p(\alpha_r^i)) > 0.$$

[15] The variance of these values as well as their correlation are irrelevant given our assumption of risk-neutrality.

The timing is as follows. Banks simultaneously decide their selling strategies so as to maximize the expected payoff as described in (1); investors assess the value of each security according to the above described procedure and form their demand as in (3); a competitive equilibrium emerges which determines the price for each security so as to clear the market.

When K is finite, the price $p(\alpha_r^i)$ may vary stochastically as a function of the profile of realizations of the assessments $\widetilde{x}_{k'}(\alpha_{r'}^{i'})$ of bundles $\alpha_{r'}^{i'}$ by the various investors k'. When we consider the case of an infinite number of investors, the randomness of prices is removed, which simplifies the derivation of some results.

Remark on Investors' Heuristic and Rationality

The heuristic of our investors is simple and we believe natural in light of the considerations discussed in the Introduction. It may be interesting to discuss which ingredients could be derived from a more orthodox approach and which ones should be attributed to bounded rationality. A first essential element is that our investors have no prior knowledge about the various assets and they assess the value of the average asset in a pool based on the signal they draw about one asset. While it is hard to derive this type of evaluation in a Bayesian framework with an objective representation of the model, one may alternatively view the investors as holding wrong perceptions about how assets are allocated into pools. If investors were to believe that assets in a pool come from i.i.d. draws say from a normal distribution, their estimate would be a convex combination between the signal they get and their prior, and if they were to attach very high precision to the signal they draw, they would behave as in our model.[16] Investors would believe their draw is representative of the average asset in the pool without considering that, in our framework, the composition of the pools is chosen by banks precisely with the desire to exploit the imperfect model employed by investors.

A second essential element is that investors only consider their own signal when forming evaluations, without trying to infer extra information through the price levels. This could be derived by allowing investors to have subjective priors for example on how information is distributed. From that perspective, our model can be viewed as considering an extreme form of overconfidence (very high weight on one own signal), which allows us to simplify (by trivializing it) the inference problem from the observation of the prices (Scheinkman and Xiong (2003)). An alternative way to limit investors' inference from prices would be to introduce

[16] This is to be contrasted with our preferred rationale in which the draw is the only basis on which the investor can base his belief (prior-free story).

noise traders in the tradition of Grossman and Stiglitz (1976).

A third ingredient is that investors only process one signal per bundle. The key feature here is that investors' sample is limited, as allowing investors to consider larger (yet limited) samples would not affect the qualitative insights developed next. Moreover, having investors who sample a limited number of signals could easily be endogenized by introducing an explicit cost of information processing. Along those lines, in the discussion section, we consider a setting in which the total number of signals each investor can draw is fixed (irrespective of the number of bundles offered by the banks) and multiple signals can be drawn from the same bundle, and we observe that our main qualitative insights hold true in these natural extensions.

3 Monopoly

We start by analyzing a monopolistic setting with $N = 1$ (we omit the superscript i for convenience). This is the simplest setting to highlight some basic insights, in particular the effect of investors' wealth on the incentives for the bank to create complex securities. Due to the sampling heuristic followed by investors, complexity creates disagreement among investors, and such a disagreement may or may not be beneficial to the bank, depending on the level of wealth in the economy. Specifically, investors' wealth determines, along the distribution of beliefs, who the marginal investor fixing the market clearing price is. The larger the wealth, the more likely it is that more optimistic investors are marginal, and so the larger is the incentive for banks to create complex securities.

3.1 Simple Securities are Optimal without Disagreement

A preliminary observation is that some disagreement among investors is needed to make it profitable for the bank to create complex securities. This can be shown most simply by considering the case of a single investor, $K = 1$.[17] Assume that the bank sells a subset $\hat{J} \subseteq J$ of its assets through simple securities (or, in other words, as separate assets). Each sold asset X_j is then correctly perceived as having value x_j, so the payoff derived by the bank from those sold assets is:

$$\min(\sum_{j\in\hat{J}} x_j, W). \tag{4}$$

[17] The setting is equivalent to one in which investors' draws are perfectly correlated and, in each bundle, the firm assigns uniform probability to which asset gets drawn.

Assume by contrast that the bank pools all its assets in a single bundle α and sells securities backed by α. A generic loan in the bundle is perceived to have value x_j with probability $1/|\alpha|$ for each $j \in \hat{J}$ and thus the payoff of the bank is:

$$\sum_{j \in \hat{J}} \frac{1}{|\alpha|} \min(|\alpha| x_j, W).$$

Such a payoff cannot strictly exceed the payoff in (4) due to the concavity of $\min(., w)$ and Jensen's inequality. The argument extends to any other partition, as reported in the following proposition.

Proposition 1 *Suppose $K = 1$. Irrespective of W, the monopolistic bank prefers to sell its assets (possibly a subset of them) separately.*

3.2 Complex Securities are Optimal with Disagreement and Enough Wealth

Another immediate observation is that complex securities cannot be profitable if the aggregate wealth W falls short of the fundamental value of those assets which are sold in the market, since selling assets separately exhausts the entire wealth and no other strategy can do better. To see this most simply, suppose that $\tau = 0$ so that all assets are optimally put for sale in the market.[18] Define

$$W_0 = \sum_{j \in J} x_j, \tag{5}$$

and suppose that $W \leq W_0$. Selling assets separately gives W and the payoff from any bundling cannot exceed W and can sometimes fall short of W due to the possibly pessimistic assessment of the bundle. We have the following:

Proposition 2 *Suppose $K > 1$, $\tau = 0$ and $W \leq W_0$. There is no bundling which strictly dominates full separation.*

More generally, bundling is used in order to extract a larger share of wealth from investors, taking advantage of the possibly optimistic assessments of the values of bundles. Building on this idea, we establish that if there are at least two investors, one sufficiently rich and another not too poor, then full separation is strictly dominated by full bundling.

[18] One can apply the same logic more generally to the subset of assets which are sold in the market.

Proposition 3 *Suppose $K > 1$ and there exist two investors $k_1 \neq k_2$ such that $w_{k_1} > Jx_J$ and $w_{k_2} > Jx_1$. Then bundling all assets into one package strictly dominates full separation.*

To show the above result, recall that by definition $x_J = \max_j x_j$ and $x_1 = \min_j x_j$. The condition $w_{k_1} > Jx_J$ ensures that full bundling delivers at least the same payoff as full separation. For each x_j, investor k_1 can pay Jx_j when sampling an asset with value x_j. Hence, irrespective of other investors' wealth, the expected payoff for the bank is at least $\sum_j \frac{1}{J} Jx_j$. If in addition we have $w_{k_2} > Jx_1$, then full bundling strictly dominates full separation. When investor k_1 draws x_1 and investor k_2 draws $x_j \neq x_1$ (which occurs with strictly positive probability) investor k_2 drives the price of the bundle strictly above Jx_1. Hence, the expected payoff from full bundling exceeds $\sum_j x_j$, that is the payoff from full separation.

Following on the same logic, we show that if there are at least two investors with sufficiently large wealth, bundling all assets into one package is optimal:

Proposition 4 *Suppose $K > 1$ and $w_k > Jx_J$ for all k. Then bundling all assets into one package strictly dominates any other strategy.*

The above result can be understood as follows (we refer to the Appendix for the complete proof). When investors are very wealthy, the price of any security is determined by the investors with the most optimistic evaluation of the corresponding bundle -that is, by the maximum of the draws across investors- irrespective of the bank's bundling strategy. When a single asset is added to a bundle α_r, it increases the dispersion in investors' evaluations about α_r. This increases the payoff of the bank, since the maximal evaluation of the bundle does not decrease and more assets are sold at such maximal evaluation. Hence, putting all assets into one bundle is the optimal strategy.

More generally, the above result highlights a basic trade-off faced by the bank. Disagreement among investors is profitable to the extent that only the investors who overvalue the bundle (as compared with the fundamental value) are willing to buy. The question then is whether the wealth possessed by those investors is sufficient to satisfy the corresponding market clearing conditions at such high prices. Asset complexity has then two effects: on the one hand, a lower fraction of investors are willing to buy, which all else equal induces lower prices. On the other hand, more wealth is extracted from those investors who are willing to buy. In the extreme case described in Proposition 4, only the most optimistic investors buy (because their wealth is very large) so that it is optimal for the bank to create as much disagreement as possible. This is attained by pooling all assets in a single bundle.

We now consider intermediate levels of wealth so as to highlight more generally how investors' wealth affects the incentives to increase the belief dispersion, which in turns determines the optimal form of securitization. We consider the simplest setting for this purpose, one with three assets and a continuum of investors.[19]

To illustrate how market clearing prices are set, suppose the bank creates a single bundle consisting of all three assets $\{X_1, X_2, X_3\}$. A fraction 1/3 of investors samples X_1 and assesses that on average assets in the bundle have value x_1; 1/3 of investors assesses the average asset as x_2, and 1/3 of investors assesses the average asset as x_3. If $W/3$ exceeds $3x_2$, the most optimistic investors drive the price to a level at which no other investor is willing to buy. In that case, the payoff of the bank is $\min(W/3, 3x_3)$. When $W/3$ is slightly lower than $3x_2$, prices are such that also investors who draw x_2 are willing to buy. Hence, the bank gets $\min(2W/3, 3x_2)$. When $W/3$ is slightly lower than $3x_1$, also investors who draw x_1 are willing to buy and the bank gets $\min(W, 3x_1)$.

It is clear that when W is sufficiently large, full bundling dominates any other strategy as it allows to sell assets as if they all had value x_3. We now characterize more precisely the optimal selling strategy -referred to as partition α^*- as a function of W. We consider here the case $\tau = 0$; the logic for $\tau = 1$ is very similar and reported in Section 5.

Proposition 5 *Suppose $K \to \infty$, $J = 3$ and $\tau = 0$. Then*

$$\alpha^* = \begin{cases} \alpha_1 = \{X_1, X_2, X_3\} & \textit{for} \quad W \geq 6x_3 + 3x_2 \\ \alpha_1 = \{X_1, X_3\},\ \alpha_2 = \{X_2\} & \textit{for} \quad W \in [2x_3 + 2x_2, 6x_3 + 3x_2) \\ \alpha_1 = \{X_1, X_2\},\ \alpha_2 = \{X_3\} & \textit{for} \quad W \in [2x_1 + 2x_2, 2x_3 + 2x_2) \\ \alpha_1 = \{X_1\},\ \alpha_2 = \{X_2\},\ \alpha_3 = \{X_3\} & \textit{for} \quad W < 2x_1 + 2x_2. \end{cases}$$

The above proposition highlights a key mechanism in our model. The larger the aggregate wealth W, the more profitable it is to create bundles with several assets of heterogeneous value. As W decreases, the bank prefers to bundle fewer assets and assets of more similar value. When wealth is sufficiently large, full bundling is optimal. The next optimal bundling is $\{X_1, X_3\}$ followed then by $\{X_1, X_2\}$ up to the point where it is best to sell assets separately.

[19] Our results with a continuum of investors hold irrespective of how wealth is distributed across investors.

Moreover, in the three asset case, we observe that if wealth is so low that pooling $\{X_1, X_2\}$ is dominated by offering $\{X_1\}, \{X_2\}$ separately, then no other bundling can be profitable. In the next proposition, we show that this insight carries over more generally whatever the number of assets.

Proposition 6 *Suppose $K \to \infty$ and $J > 1$ and $\tau = 0$. Some bundling dominates full separation if and only if $W > 2(x_2 + x_1)$.*

3.3 Heterogeneity and Bundle Composition

Propositions 3 and 4 show when pooling all assets in a single bundle is profitable. In the following proposition, we consider the case in which several assets have the same value, and we observe that within homogeneous compositions, it is best to create bundles which are as small as possible.

Proposition 7 *Suppose $K \to \infty$ and the bank has $2\chi_1$ assets Y with value 0 and $2\chi_2$ assets Z with value $z > 0$, where χ_1 and χ_2 are positive integers. Then creating a single bundle $\{2\chi_1 Y, 2\chi_2 Z\}$ is dominated by creating two bundles, each with $\{\chi_1 Y, \chi_2 Z\}$.*

The intuition behind the result is simple, and can be illustrated when $\chi_1 = \chi_2 = 1$. If the monopolist pools all its assets in the bundle $\{Y, Y, Z, Z\}$, its payoff is $\min(4z, W/2)$ since the maximal wealth that can be extracted comes from investors making a Z draw, i.e., half the population of investors. By creating two bundles $\{Y, Z\}, \{Y, Z\}$, its payoff is $\min(4z, 3W/4)$ where the term $3W/4$ accounts for the fact that an investor making a good draw from either bundle (there are $3/4$ of them) is potentially willing to put his wealth in the market. By disaggregating, the monopolist does not affect the probability of inducing over evaluations of the bundles since the composition of each bundle remains the same. But disaggregating allows the monopolist to extract more wealth since it reduces the fraction of investors who end up with bad draws from all bundles. A similar effect occurs as one increases the number of banks (and so possibly of bundles) in the market, which we consider in the next oligopoly section.

4 Oligopoly

We now consider a setting with several banks and analyze the effect of competition. We first look at the incentives to create complex securities as a function of aggregate wealth. Relative to the monopoly case, we find that competition typically increases such incentives. We then show that complex securities create a negative externality on the other banks, in the sense that -irrespective of its strategy- each bank is better off if the other banks offer simple securities. This externality leads to a new phenomenon: offering complex securities can be the only equilibrium and at the same time be detrimental to banks, in the sense that banks would be better off if they could coordinate on a different strategy. This is similar to the so called prisoner's dilemma in game theory.

We wish to present our results in the simplest setting. We consider a market with $N \geq 3$ banks and a continuum of investors, we assume that $\tau = 0$ and each bank has only two assets, with $x_1^i = 0$ and $x_2^i = x_2 > 0$ for all i. The selling strategy for each bank is simple. It can either offer complex securities backed by the pool $\{X_1, X_2\}$ (or, in short, it can bundle), or simple securities $\{X_1\}, \{X_2\}$ (that is, sell assets separately). We consider richer settings in Section 5.

4.1 Complexity and Wealth

In order to see when bundling can be sustained in equilibrium, suppose all banks offer securities backed by the pool $\{X_1, X_2\}$. Investors who sample x_2 from at least one pool are attracted in the market. Their aggregate wealth is $(1-(\frac{1}{2})^N)W$, which is equally shared among the N banks. The payoff for each bank is thus:

$$\pi^B = \min(2x_2, \frac{1}{N}(1-(\frac{1}{2})^N)W). \tag{6}$$

Suppose now that one bank, say bank d, sells its assets separately while other banks continue to pool their assets. The payoff of the deviating bank d is

$$\pi^{-B} = \min(x_2, \frac{1}{2N-1}W). \tag{7}$$

To derive expression (7), we observe that market clearing requires that investors sampling x_2 from a bundle α are indifferent between buying the security backed by α and the single asset X_2^d. If investors were strictly preferring X_2^d, no one would be willing to buy the bundle. If investors were strictly preferring the security over the separate asset X_2^d, only investors sampling x_1 from all bundles would buy X_2^d. When N is large (as it turns out $N \geq 3$ is enough for this), that is a small fraction of investors and so the price of X_2^d would be too low to induce investors to buy the security.[20] Hence, asset X_2^d of the deviating bank gets pooled with all the bundles,

[20] The case when $N = 2$ is discussed in Section 5.

and the market behaves as one with $(2N-1)$ identical assets with value x_2. The deviating bank then attracts a fraction $1/(2N-1)$ of the aggregate wealth and, given that the price of X_1^d is unambiguously 0, that leads to the payoff shown in equation (7). From equations (6) and (7), we see that bundling can be sustained in equilibrium for all levels of W given that $\pi^B > \pi^{-B}$ for all W. This is summarized in the following proposition.

Proposition 8 *Bundling is an equilibrium for all W.*

Comparing with the monopoly case, in which the monopolist bundles when $W > 2x_2$, we observe that bundling occurs even for levels of wealth at which a monopolist would not bundle. In an oligopolistic setting in which all banks bundle, it would be too costly to deviate and sell assets separately. Separate assets of value x_2 would be considered as equally attractive as the bundles and so the deviating bank would receive a lower fraction of investors' wealth.

4.2 The Externality of Complex Securities

An important effect of competition comes from the fact that offering complex securities creates a negative externality on the other banks. Irrespective of its strategy, each bank is better off if the other banks offer simple securities. The reason is that when a bank creates complex securities it attracts more wealth from those having a positive evaluation of the bundle, which in turn reduces the amount of wealth available to buy the other assets and that is detrimental to the other banks. To see this more precisely, observe that if all banks sell the assets separately their payoff is

$$\pi^U = \min(x_2, \frac{W}{N}). \tag{8}$$

If bank d deviates and offers the complex security while other banks sell their asset separately, the payoff of bank d is

$$\pi^{-U} = \min(2x_2, \frac{2}{N+1}W). \tag{9}$$

The logic behind equation (9) is similar to that behind (7). Market clearing requires that investors who sample x_2 from the bundle are indifferent between the complex security and any asset X_2 of the non-bundling banks, so that the market behaves as one with $N+1$ assets of value x_2. The externality of bundling can be expressed as follows:

Proposition 9 *Irrespective of its strategy, each bank is better off when all other banks offer the assets separately than when they bundle. That is,* $\pi^U \geq \pi^{-B}$ *and* $\pi^{-U} \geq \pi^B$ *for all* W *where* π^U, π^{-B}, π^{-U} *and* π^B *are defined respectively in (8), (7), (9) and (6).*

4.3 The Curse of Complexity

The above described externality leads to a new phenomenon, which we call Cursed Bundling: offering complex securities can be the only equilibrium and at the same be collectively bad for banks, in the sense that if banks could make a joint decision they would rather offer simple securities.

Definition 1 *We have Cursed Bundling when three conditions are met: i) Bundling is an equilibrium; ii) Banks would be better off by jointly deciding not to bundle; iii) Not bundling is not an equilibrium.*

As shown in Proposition 8, no restriction on W is required to meet condition (i). Condition (ii) requires that W is not too large, or bundling would not be dominated. In particular, since by jointly deciding not to bundle each bank gets π^U as defined in (8), condition (ii) requires that $\pi^B < x_2$. Given π^B in (6), that defines an upper bound on wealth, i.e. $W \leq W_1$ where

$$W_1 = \frac{N}{1 - (\frac{1}{2})^N} x_2. \tag{10}$$

Condition (iii) instead is never binding. To see this, notice that bundling displays a form of strategic substitutability in the sense that each bank's incentive to bundle (compared to not bundling) is higher when the other banks do not bundle. By not bundling, the other banks are likely to extract a lower fraction of wealth from investors, and the incentives to bundle increase with the amount of wealth which remains to be extracted. Formally, we have:

Lemma 1 *The incentive to bundle is larger when the other banks do not bundle. That is,* $\pi^B - \pi^{-B} \leq \pi^{-U} - \pi^U$ *for all* W*.*

The previous lemma implies that if bundling is an equilibrium, then selling assets separately is not an equilibrium. Hence, in Definition 1, condition (iii) is implied by condition (i). Taken together, the above considerations imply that we have cursed bundling when W is no larger than W_1, as summarized in the following proposition.

Proposition 10 *Cursed Bundling occurs for all $W < W_1$.*

An interesting observation is that in cursed bundling, the mere option of banks to offer complex securities, together with investors' inability to correctly assess the values of the bundles, makes investors better off. Moreover, for $W \in (Nx_2, W_1)$, prices would be equal to fundamentals if banks offered simple securities and turn out to be below fundamentals only due to bundling.

In the basic setup considered up to now, cursed bundling requires that equilibrium prices are below fundamentals (as implied by $\pi^B < x_2$). Such a property is however not required in more general specifications, as illustrated in the next section.

Moreover, by assuming $\tau = 0$, we ruled out the possibility that banks may withhold some of their assets. This is relaxed in the next section, which allows to highlight how cursed bundling can prevent banks and investors from fully exploiting potential gains from trade.

5 Robustness

We have so far analyzed the simplest settings needed to develop our insights. We now address the robustness of our results and discuss the effect of having $\tau \neq 0$, a small number of investors and two banks. This section considers various settings within the baseline model of Section 2. Extensions are considered in Section 6.

5.1 When Banks Withhold their Assets

Suppose that banks have no fundamental reasons to sell their assets. That is, $\tau = 1$. The insights we have developed in the monopolistic setting with $\tau = 0$ would not be substantially changed. As shown in the Appendix, Proposition 5 would hold except that when the bank would sell an asset X_j separately in the context of Proposition 4 (i.e. whenever $\tau = 0$), it now withholds it, and Proposition 6 would be unchanged.

A general property that holds beyond the three asset case is that, conditionally on selling something, the monopolist would always include its worst asset in a bundle when $\tau = 1$. To see this, suppose by contradiction that X_1 is kept in the bank. As we know, bundling is profitable only if the price does not depend on the lowest possible evaluation (the lowest X_w in the bundle). Replacing the worst asset X_w in the bundle with X_1 would not affect the price of the corresponding security (nor of any other security possibly offered by the bank), but would improve the payoff of the bank as the bank can keep X_w instead of X_1. This is summarized in the following proposition.

Proposition 11 *Suppose that $\tau = 1$, and the monopolistic bank puts at least one asset in the market. Then it necessarily puts the asset with lowest return X_1 in the market.*

Turning to an oligopolistic setting, observe that when $\tau = 1$ and a bank withholds its assets, it gets the fundamental values independently of what other banks do. It follows that, in equilibrium, bundling must give to each bank at least such fundamental values. That is also the maximal payoff obtained if there were no bundle in the economy, and so cursed bundling in the strong sense defined previously cannot arise when $\tau = 1$.

Interestingly however, one can still have that banks are induced for strategic reasons to create "excessive" complexity as compared to what would be collectively optimal for the banks. The simplest setting to illustrate the possibility of cursed bundling in this broader sense is one with $N \geq 3$ banks, each of them with two assets valued $x_1 = 0$ and one asset valued $x_2 > 0$. Full bundling occurs when each bank pools all its assets in a single bundle. We say there is partial bundling when each bank keeps one asset with value x_1 and pools the other assets (with value x_1 and x_2) in a single bundle. We modify the definition of cursed bundling as follows.

Definition 2 *We have Partially Cursed Bundling when i) Full bundling is an equilibrium; ii) Full bundling is dominated by partial bundling; iii) Partial bundling is not an equilibrium.*

If all banks create the full bundle, the payoff of each bank is

$$\pi^F = \min(3x_2, \frac{1}{N}(1 - (\frac{2}{3})^N)W), \tag{11}$$

where the term $\frac{1}{N}(1-(\frac{2}{3})^N)W$ accounts for the total wealth of the share of investors who make a positive draw X_2 for at least one of the N bundles. Full bundling can be sustained in equilibrium only if $\pi^F \geq x_2$, since each bank can get x_2 by withholding its assets. That requires $W \geq W_2$, where

$$W_2 = \frac{N}{1 - (\frac{2}{3})^N} x_2. \tag{12}$$

As we show in the next proposition (whose proof follows the logic discussed in the previous section and is developed in the Appendix) full bundling is indeed an equilibrium for $W \geq W_2$. As for condition (ii), if banks could jointly follow the partial bundling strategy, their payoff would be

$$\pi^P = \min(2x_2, \frac{1}{N}(1 - (\frac{1}{2})^N)W). \tag{13}$$

Full bundling is dominated by partial bundling when $\pi^F < 2x_2$. That is, when $W < 2W_2$. Finally, similarly to the previous section, one can show that condition (iii) never binds: if full bundling is an equilibrium, then partial bundling is not an equilibrium. This leads to the following proposition.

Proposition 12 *There is Partially Cursed Bundling for* $W \in (W_2, 2W_2)$.

Partially cursed bundling requires that W is not too large, or full bundling would be the optimal strategy. It also requires that W is not too small, or else banks would prefer to withhold their assets. (The latter is different from what we had in the previous subsection when $\tau = 0$.) As it turns out, under the conditions of Proposition 12, when there is partially cursed bundling with $\tau = 1$, it is also so with $0 \leq \tau < 1$, the reason being that deviations are less profitable with $0 \leq \tau < 1$ than with $\tau = 1$.[21]

Remark on Cursed Bundling and Welfare

The previous analysis has focused on the pricing implications of cursed bundling. It is clear however that in general the possibility of cursed bundling affects also the range of assets that banks decide to sell in the market. Unless $\tau = 0$, lower prices induced by cursed bundling may lead some banks to not sell all their assets. Unless $\tau = 1$, such lower supply implies that profitable gains from trade between banks and investors cannot be exploited. Such missed trading opportunities would induce a welfare loss, as a result of cursed bundling.

5.2 Small Number of Investors

In the limit of a very large number of investors, the bank does not face any uncertainty on the payoff associated to a given bundling strategy. The reason is that the belief of the marginal investor (that is, the lowest evaluation among those investors who are willing to buy a given bundle) is deterministic. Having a finite number of investors introduces a (mean-preserving) spread on such beliefs. This may increase or decrease the payoff that results from the bundling strategy,

[21] Note however that we may have partially cursed bundling with $\tau < 1$ and not with $\tau = 1$.

depending on whether such a payoff is concave or convex in the marginal belief. This in turn depends on the aggregate wealth W. Consider a setting with two assets, and denote by π_K^B the expected payoff obtained by the monopolist from bundling when there are K equally wealthy investors and by π_∞^B the corresponding payoff when $K \to \infty$.

Remark 1 *Suppose $N = 1$, $J = 2$ and $w_K = W/K$ for all K. If $W \in (2x_1, 4x_1]$, then $\pi_K^B > \pi_\infty^B$ for all K. If $W \geq 4x_2$, then $\pi_K^B < \pi_\infty^B$ for all K.*

When $W \in (2x_1, 4x_1]$ and $K \to \infty$, the price of the complex security cannot exceed $2x_1$. The marginal investor is the one with the lowest evaluation in the population, so there is nothing to lose but possibly something to gain from a spread in beliefs. Conversely, when $W \geq 4x_2$ and $K \to \infty$, the price of the complex security is for sure $2x_2$. The marginal investor is the one with the highest valuation of the asset, so there is nothing to gain but possibly something to lose when K is finite.[22]

A setting with finite K also reveals that, while bundling can be profitable ex-ante, it can be detrimental to the bank ex-post. Bad evaluations may be higher than expected and so the marginal belief may be lower than expected. Similar insights can be derived by introducing stochastic W, which we consider in the next section.

5.3 Two Banks

In Section 4, we considered a setting with $N \geq 3$ banks so as to simplify the exposition. When $N = 2$, the equilibrium is slightly different. As in Section 4, assume a continuum of investors, $\tau = 0$ and $x_1^i = 0$ and $x_2^i = x_2 > 0$ for all i but let $N = 2$. Suppose both banks bundle and bank d deviates and sell its assets separately. Differently from the case in which N is large, investors who sample x_2 from the bundle strictly prefer buying the complex security as opposed to X_2^d. The deviating bank attracts only investors who sample x_1 from the other bank, that corresponds to half of the aggregate wealth. Its payoff is then $\min(W/2, x_2)$. The deviation is profitable unless $\pi^B \geq x_2$. Given (6) at $N = 2$, $\pi^B \geq x_2$ requires $W \geq 8x_2/3$. It follows that bundling is an equilibrium when W is large enough,

[22] An additional source of uncertainty arises if investors have heterogeneous wealth. Again, that may increase or decrease the payoff from bundling depending on whether wealth is low or high. In a setting with $J = K = 2$, for example, having one investor with wealth $w - \varepsilon$ and the other investor with wealth $w + \varepsilon$, as opposed to two investors with w, increases payoffs when $W \in (2x_1, 4x_1]$ and it decreases payoffs when $W \geq 4x_2$.

and that cursed bundling cannot occur. In fact, when W is small, the payoff from bundling cannot be smaller than $W/2$, which is the maximum payoff from not bundling. This is summarized in the following proposition.

Proposition 13 *If $N = 2$, bundling is an equilibrium for $W \geq 8x_2/3$. We have no Cursed Bundling.*

6 Extensions

In this section, we discuss a number of extensions of our basic model. First, we consider the possibility of short selling. Then, we consider alternative evaluation procedures investors may employ: we assume they can draw a fixed number of signals; they can sample more assets from each bundle; they can discount bundles for complexity motives. We then discuss the possibility of tranching and the role of risk aversion in our setting. Finally, we consider the case of stochastic wealth.

6.1 Short Selling

Introducing short-selling has the obvious effect that investors with low evaluations can drive the price down. A less obvious effect is that short selling may increase the incentives to create disagreement by offering complex securities in a competitive setting: while short selling decreases the payoff from bundling, it also decreases the payoff from deviations. Separate assets attract a lower amount of wealth since investors may use their wealth to short sell the complex securities. We show in a simple example that the second effect may be stronger, thereby making both bundling and cursed bundling more likely to occur.

Suppose that investors can short sell and that short selling is limited by capital constraints (say due to collateral requirements). Specifically, suppose that an investor with wealth w can short sell $\theta w/p$ units of an asset of price p (the baseline model corresponds to $\theta = 0$). Consider a setting with $N = 2$ banks each with two assets with value $x_1 = 0$ and $x_2 > 0$, $\tau = 0$ and a continuum of investors.

Suppose both banks bundle and denote by p^B the price of the corresponding security. An investor drawing x_1 from one bundle and x_2 from another bundle prefers to buy rather than short selling if

$$p^B \leq \frac{x_2}{1+\theta}.$$

As we show in the Appendix, bundling can be sustained in equilibrium when W is sufficiently large, and the required lower bound on wealth is defined by

$$\frac{3-\theta}{8}W_3 = \frac{x_2}{1+\theta}. \tag{14}$$

In equation (14), the l.h.s. is the payoff from bundling and the r.h.s. is the payoff from deviation. An interesting observation is that both payoffs decrease with θ, but the payoff from deviation decreases more. This implies that W_3 decreases in θ for $\theta < 1$. Hence, bundling is more likely to occur when $\theta > 0$ than when no short selling is allowed.

A corollary is that short selling creates the possibility of cursed bundling, which was not possible with $N = 2$ and $\theta = 0$. The payoff from bundling is lower than what banks would get by jointly deciding not to bundle whenever

$$\frac{3-\theta}{8}W < x_2.$$

That is, when $W < W_3(1+\theta)$. We then have the following.[23]

Proposition 14 *Suppose that $N = 2$ and short selling is allowed.*
i) Bundling is an equilibrium iff $W \geq W_3$.
ii) Cursed Bundling occurs for $W \in (W_3, W_3(1+\theta))$.

6.2 Varying the Sophistication of Investors

A natural way in which one could modify the above setting is by letting investors make several draws as opposed to one to assess the value of bundles. Clearly, if an investor makes a very large number of draws, in the limit he has the correct evaluation of bundles, and if all investors do that, there is no point for banks in bundling. But suppose that each investor has a fixed number of signals M he can draw, irrespective of banks' securitization strategies, and M is strictly lower than the total number of assets in the economy. Suppose also that if an investor does not make any draw from bundle α_r^i the investor does not buy α_r^i irrespective of the price. A discussion of the robustness of our main findings follows.

First observe in the monopoly case that when full bundling was optimal in the main analysis (i.e. for W large enough) it remains so when $M = 1$ given that under the extra constraint imposed by $M = 1$ the payoff to the bank is the same

[23] It can also be shown, as in the previous section, that if bundling is an equilibrium then no bundling is not an equilibrium.

in the full bundling scenario and can only be lower in other bundling scenarios. Similar insights as in the main analysis would also arise in the monopoly case even as M is increased.

To discuss oligopolistic structures, we consider the simplest setting with $J = 2$, $x_1 = 0$ and $x_2 > 0$, $\tau = 0$ and $K \to \infty$ and we assume $N = M = 2$. Suppose first that investors sample each bundle at most once (possibly due to the fact that they do not observe the size of each bundle). One can show that bundling would be an equilibrium for $W > 8x_2/3$ as in the main analysis. Suppose instead that investors observe the size of the bundle and that they can sample several assets from the same bundle (they randomly draw across all assets with no replacement). Relative to the baseline model, bundling is now more likely to occur. In the relevant range of wealth, bundling is more profitable since the fraction of investors who do not buy (those with only bad draws) is lower and at the same time deviations are less profitable as they attract lower wealth. This is shown in the next proposition.

Proposition 15 *Suppose that $N = M = 2$. Bundling is an equilibrium for all W.*

Another way in which our investors could be more sophisticated even if relying on small samples is in being aware that their assessments of the bundles are imperfect, thereby leading them to apply some discount in their evaluations. Observe however that investors in our baseline setting are not required to know the size of each bundle so in that sense they perceive all bundles as alike. If the discount is applied uniformly to all bundles, no qualitative property in our analysis would be affected.

One could instead enrich the model and allow investors to know the size of each bundle and apply a larger discount to larger bundles (since their evaluations tend to be less precise there). In the Appendix, we develop an example along these lines. We assume that banks have two assets and an investor who samples x_j from a bundle with two assets has valuation $x_j - \delta$, where δ is the discount for complexity (evaluations of single assets remain unchanged). We show that, provided that δ is not too large (essentially, $\delta < x_2 - x_1$) our previous results are not affected. The thresholds on W defined in our main analysis are determined by wealth constraints and so they are insensitive to (small) changes in investors' evaluations.

Along those lines, one could also assume that investors apply larger discounts when they perceive larger asset heterogeneity. This (immediate) extension of our model would shed light on why banks may bundle assets of very heterogeneous quality (say mortgages with diverse default probabilities) but they tend not to bundle assets of very different type (say mortgages together with car loans).

Finally, one may consider a setting with investors who are heterogeneous in their sophistication. Consider the variant in which a share β of investors is rational (they can be thought of as making infinitely many draws) and a share $1-\beta$ makes only one draw to evaluate each bundle as in the baseline model. In the context of Proposition 10 when $\tau = 0$, we would have cursed bundling exactly under the same conditions if the share β is small enough. Rational investors would still find it optimal to buy the bundle as its price is below the fundamental.[24] In the context of Proposition 12 when $\tau = 1$, rational investors would stay out of the market given that prices are above the fundamental and the condition for partially cursed bundling would have to be replaced by $(1-\beta)W \in (W_2, 2W_2)$.

6.3 Risk Aversion

Our main analysis assumes that investors are risk neutral so as to abstract from risk sharing considerations that could motivate bundling. Allowing for risk aversion in our baseline model may actually reinforce the incentives for bundling, as we now explain. Suppose our basic assets are loans with face value equal to 1 and probability of default equal to $1-x_j$. Suppose defaults are (perceived as) independent across loans. Suppose also that investors observe the size of each bundle.[25] If two assets with respective values x_2 and x_1 are offered separately, the investor perceives an expected value of $x_2 + x_1$ and a variance of $x_1(1-x_1) + x_2(1-x_2)$. If the assets are bundled, an investor drawing x_2 believes that the bundle has expected value $2x_2$ and variance $2x_2(1-x_2)$. If the investor buys $(x_2+x_1)/2x_2$ units of the bundle, he perceives the same expected value x_2+x_1, but a variance of $(1-x_2)(x_2+x_1)^2/2x_2$, which is lower than $x_1(1-x_1) + x_2(1-x_2)$. If investors dislike payoffs with larger variance, the bank has an extra incentive to bundle assets x_1 and x_2. Bundling may lead investors not only to overestimate the expected value but also to underestimate the variance of returns.

6.4 Tranching

An additional motive for bundling assets is to create different tranches which are then sold to investors with different risk appetites. In our setting, tranching can

[24] The share β has to be small enough to make the deviation of selling the assets separately undesirable.

[25] This is not needed as the argument would hold irrespective of the size of the bundle, but it simplifies the exposition.

be profitable even if investors are risk neutral. Tranching may be a way to exploit belief heterogeneity and, relative to selling pass-through securities, it may allow the bank to extract a larger share of wealth. We also show this is the case even if the bank were required to keep the most junior tranche.

To see this most simply, consider a bank ($N = 1$) with two assets with $0 < x_1 < x_2$. The bank can offer a pass-through security or slice the bundle into a junior and a senior tranche. The senior tranche pays 1 if at least one loan is repaid, the junior tranche pays 1 if both loans are repaid. Assume $K \to \infty$, investors are risk neutral and they observe the size of each bundle. Investors who sample x_1 value the senior tranche as $s_1 = 1 - (1 - x_1)^2 = 2x_1 - x_1^2$ and the junior tranche as $j_1 = x_1^2$. Similarly, investors who sample x_2 value the senior tranche as $s_2 = 2x_2 - x_2^2$, and the junior tranche as $j_2 = x_2^2$.

We provide some intuition about how the equilibrium works and refer to the Appendix for more details. Suppose the monopolist sells the two tranches and denote its payoff as π^T. At low levels of wealth, everyone buys both tranches and the payoff is W, which coincides with the payoff from offering a pass-through security π^B when $W \leq 2x_1$. When W is large, those sampling x_2 drive the prices so high that investors sampling x_1 prefer not to buy any tranche. In particular, as shown in the Appendix, this occurs when $W > 2W_4$, where

$$W_4 = \frac{s_1}{s_2}(j_2 + s_2).$$

In that case, we have again $\pi^T = \pi^B$. Tranching is however strictly preferred to the pass-through security for intermediate levels of wealth, for which investors sampling x_2 buy both tranches while those sampling x_1 only buy the senior tranche. Tranching is profitable to the bank as it allows to price discriminate between those having optimistic views and those having pessimistic views about the bundle. This is shown in the next proposition.[26]

Proposition 16 *We have $\pi^T \geq \pi^B$ for all W and $\pi^T > \pi^B$ for $W \in (2x_1, 2W_4)$.*

Finally, it should be noted that in our setting the bank would still have an incentive to securitize even if it were required to keep the most junior tranche. Indeed, if investors' wealth is large enough, the bank would still benefit from selling the senior tranche to the most optimistic investors.

[26] It should be noted though that $W_4 < (x_1 + x_2)$, so that the monopolist stills prefer selling its assets separately rather than as a bundles with two tranches when $W < 2(x_1 + x_2)$. Hence, in this example, allowing the monopolist to sell assets in tranches does not strictly improve its payoffs. We suspect it could be otherwise in more elaborate situations.

6.5 Stochastic Wealth

An interesting aspect of our analysis is that creating complex securities may sometimes result in overpricing when the economy is wealthy enough and sometimes in underpricing when there is not enough money.

Of course, in a static setting with deterministic wealth as the one considered above, banks would not create complex securities if it resulted in underpricing. But, if there were shocks on the aggregate wealth, then one may well have that in the face of uncertainty banks opt for bundling and when a negative shock on aggregate wealth happens, assets get underpriced.

This is illustrated through the following simple example. Consider the monopoly case with two assets having respective values $0 < x_1 < x_2$, $\tau = 0$, and $K \to \infty$. Aggregate wealth W can take value $\underline{W}$ with probability $1/3$ and $\overline{W}$ with probability $2/3$, where $\underline{W} \in (x_1 + x_2, 4x_1)$, $\overline{W} \in (4x_2, +\infty)$ and $3x_1 > x_2$.

If the two assets are sold separately, the bank gets the fundamental values $x_1 + x_2$ irrespective of the realization of W since $\underline{W} > x_1 + x_2$. Assume now that the bank sells the complex securities. When $W = \underline{W}$ realizes, its payoff is $2x_1$, the price is determined by the pessimists. When $W = \overline{W}$ realizes, the payoff is $2x_2$, the price is determined by the optimists. The expected payoff of the bank is $\frac{2}{3}x_1 + \frac{4}{3}x_2$, which is bigger than the payoff obtained without bundling. Thus, the bank optimally chooses to create complex securities, and as compared with the fundamental, there is overpricing when $W = \overline{W}$ and underpricing when $W = \underline{W}$.

Extending our framework so as to incorporate the dynamics of wealth across investors is in our view an interesting avenue for future research.

References

Adams, W. J. and Yellen, J. L. (1976), 'Commodity bundling and the burden of monopoly', *Quarterly Journal of Economics* **90**(3), 475–498.

Arora, S., Barak, B., Brunnermeier, M. and Ge, R. (2011), 'Computational complexity and information asymmetry in financial products', *Communications of the ACM* **54**(5), 101–107.

Baquero, G. and Verbeek, M. (2008), 'Do sophisticated investors believe in the law of small numbers?', Mimeo.

Barberis, N., Shleifer, A. and Vishny, R. (1998), 'A model of investor sentiment', *Journal of Financial Economics* **49**(3), 307–343.

Benartzi, S. (2001), 'Excessive extrapolation and the allocation of 401(k) accounts to company stock', *Journal of Finance* **56**(5), 1747–1764.

Bernardo, A. E. and Cornell, B. (1997), 'The valuation of complex derivatives by major investment firms: Empirical evidence', *Journal of Finance* **52**(2), 785–798.

Biais, B. and Mariotti, T. (2005), 'Strategic liquidity supply and security design', *Review of Economic Studies* **72**(3), 615–649.

Bianchi, M. and Jehiel, P. (2015), 'Financial reporting and market efficiency with extrapolative investors', *Journal of Economic Theory* **157**, 842–878.

Bolton, P., Freixas, X. and Shapiro, J. (2012), 'The credit ratings game', *Journal of Finance* **67**(1), 85–111.

Broer, T. and Kero, A. (2016), 'Collateralized lending and asset prices when investors disagree about risk'.

Carlin, B. I., Longstaff, F. A. and Matoba, K. (2014), 'Disagreement and asset prices', *Journal of Financial Economics* **114**(2), 226–238.

Célérier, C. and Vallée, B. (2015), 'What drives financial complexity? a look into the retail market for structured products'.

Chen, J., Hong, H. and Stein, J. C. (2002), 'Breadth of ownership and stock returns', *Journal of Financial Economics* **66**(2), 171–205.

Choudry, M. and Buchanan, B. (2014), 'Back to the future: 900 years of securitization', *Journal of Risk Finance* **15**(4), 316–333.

Coval, J., Jurek, J. and Stafford, E. (2009), 'The economics of structured finance', *Journal of Economic Perspectives* **23**(1), 3–25.

De Long, J. B., Shleifer, A., Summers, L. H. and Waldmann, R. J. (1990), 'Positive feedback investment strategies and destabilizing rational speculation', *Journal of Finance* **45**(2), 379–395.

DeMarzo, P. (2005), 'The pooling and tranching of securities: A model of informed intermediation', *Review of Financial Studies* **18**(1), 1–35.

DeMarzo, P. and Duffie, D. (1999), 'A liquidity-based model of security design', *Econometrica* **67**(1), 65–99.

Dominitz, J. and Manski, C. F. (2011), 'Measuring and interpreting expectations of equity returns', *Journal of Applied Econometrics* **26**(3), 352–370.

Downing, C., Jaffee, D. and Wallace, N. (2009), 'Is the market for mortgage-backed securities a market for lemons?', *Review of Financial Studies* **22**(7), 2457–2494.

Efing, M. and Hau, H. (2015), 'Structured debt ratings: Evidence on conflicts of interest', *Journal of Financial Economics* **116**(1), 46–60.

Farhi, E. and Tirole, J. (2015), 'Liquid bundles', *Journal of Economic Theory* **158**, 634–655.

Fratianni, M. (2006), 'Government debt, reputation and creditors' protections: The tale of san giorgio', *Review of Finance* **10**(4), 487–506.

Frehen, R., Goetzmann, W. N. and Rouwenhorst, K. G. (2013), Dutch securities for american land speculation in the late eighteenth century, *in* 'Housing and Mortgage Markets in Historical Perspective', University of Chicago Press, pp. 287–304.

Furfine, C. H. (2014), 'Complexity and loan performance: Evidence from the securitization of commercial mortgages', *Review of Corporate Finance Studies* **2**(2), 154–187.

Gennaioli, N., Shleifer, A. and Vishny, R. (2012), 'Neglected risks, financial innovation, and financial fragility', *Journal of Financial Economics* **104**(3), 452–468.

Ghent, A. C., Torous, W. N. and Valkanov, R. I. (2014), 'Complexity in structured finance: Financial wizardry or smoke and mirrors?'.

Gorton, G. and Metrick, A. (2012), 'Securitization'.

Greenwood, R. and Nagel, S. (2009), 'Inexperienced investors and bubbles', *Journal of Financial Economics* **93**(2), 239–258.

Greenwood, R. and Shleifer, A. (2014), 'Expectations of returns and expected returns', *Review of Financial Studies* **27**(3), 714–746.

Grossman, S. J. and Stiglitz, J. E. (1976), 'Information and competitive price systems', *American Economic Review* **66**(2), 246–253.

Harrison, J. M. and Kreps, D. M. (1978), 'Speculative investor behavior in a stock market with heterogeneous expectations', *Quarterly Journal of Economics* **92**(2), 323–36.

Henderson, B. J. and Pearson, N. D. (2011), 'The dark side of financial innovation: A case study of the pricing of a retail financial product', *Journal of Financial Economics* **100**(2), 227–247.

Jehiel, P., Meyer-Ter-Vehn, M. and Moldovanu, B. (2007), 'Mixed bundling auctions', *Journal of Economic Theory* **134**(1), 494–512.

Krugman, P. (2007), 'Innovating our way to financial crisis'.

Lewis, M. (2010), *The Big Short: Inside the Doomsday Machine*, W.W. Norton.

Mason, J. R. and Rosner, J. (2007), 'Where did the risk go? how misapplied bond ratings cause mortgage backed securities and collateralized debt obligation market disruptions'.

McAfee, R. P., McMillan, J. and Whinston, M. D. (1989), 'Multiproduct monopoly, commodity bundling, and correlation of values', *Quarterly Journal of Economics* **104**(2), 371–383.

Myers, S. and Majluf, N. (1984), 'Corporate financing and investment decisions when firms have information that investors do not have', *Journal of Financial Economics* **13**(2), 187–221.

Opp, C. C., Opp, M. M. and Harris, M. (2013), 'Rating agencies in the face of regulation', *Journal of Financial Economics* **108**(1), 46–61.

Pagano, M. and Volpin, P. (2012), 'Securitization, transparency, and liquidity', *Review of Financial Studies* **25**(8), 2417–2453.

Palfrey, T. R. (1983), 'Bundling decisions by a multiproduct monopolist with incomplete information', *Econometrica* **51**(2), 463–83.

Rabin, M. (2002), 'Inference by believers in the law of small numbers', *Quarterly Journal of Economics* **117**(3), 775–816.

Rabin, M. and Vayanos, D. (2010), 'The gambler's and hot-hand fallacies: Theory and applications', *Review of Economic Studies* **77**(2), 730–778.

Rajan, U., Seru, A. and Vig, V. (2015), 'The failure of models that predict failure: Distance, incentives, and defaults', *Journal of Financial Economics* **115**(2), 237–260.

Riddiough, T. J. and Thompson, H. E. (2012), 'Déjà vu all over again: Agency, uncertainty, leverage and the panic of 1857'.

Sato, Y. (2014), 'Opacity in financial markets', *Review of Financial Studies* **27**(12), 3502–3546.

Scheinkman, J. A. and Xiong, W. (2003), 'Overconfidence and speculative bubbles', *Journal of Political Economy* **111**(6), 1183–1219.

Shiller, R. J. (2000), 'Measuring bubble expectations and investor confidence', *Journal of Psychology and Financial Markets* **1**(1), 49–60.

Simsek, A. (2013), 'Speculation and risk sharing with new financial assets', *Quarterly Journal of Economics* **128**(3), 1365–1396.

Skreta, V. and Veldkamp, L. (2009), 'Ratings shopping and asset complexity: A theory of ratings inflation', *Journal of Monetary Economics* **56**(5), 678–695.

Soros, G. (2009), *The Crash of 2008 and What It Means: The New Paradigm for Financial Markets*, PublicAffairs.

Spiegler, R. (2006), 'The market for quacks', *Review of Economic Studies* **73**(4), 1113–1131.

Tversky, A. and Kahneman, D. (1971), 'Belief in the law of small numbers', *Psychological Bulletin* **76**(2), 105 – 110.

Tversky, A. and Kahneman, D. (1975), Judgment under uncertainty: Heuristics and biases, *in* 'Utility, probability, and human decision making', Springer, pp. 141–162.

Xiong, W. (2013), Bubbles, crises, and heterogeneous beliefs, *in* 'Handbook on Systemic Risk', Cambridge University Press, pp. 663–713.

Zandi, M. (2008), *Financial Shock: A 360-degree Look at the Subprime Mortgage Implosion, and How to Avoid the Next Financial Crisis*, FT Press.

7 Appendix

7.1 Omitted Proofs

Proof of Proposition 1

Denote with $\alpha = \{\alpha_r\}_r$ with $r = 1, 2, ..., R$ an arbitrary partition of the $\hat{J}$ assets. We denote with x^r a generic element of bundle α_r and with $y = (x^1, x^2, .., x^R)$ a generic vector in which one asset of bundle α_1 is associated with one asset from each of the bundles $\alpha_2, \alpha_3, .., \alpha_R$. We denote with Y the set of all possible vectors y. The payoff from selling assets with partition α is $\pi(\alpha) = \sum_{y \in Y} \eta(\alpha) \min(W, \pi_0(y))$, where $\eta(\alpha) = \prod_r \frac{1}{|\alpha_r|}$, and $\pi_0(y) = \sum_{x^r \in y} |\alpha_r| \, x^r$. Notice that by definition $\sum_{y \in Y} \eta(\alpha) = 1$ and so $\pi(\alpha) \leq W$. Notice also that $\pi(\alpha) \leq \sum_{j \in \hat{J}} x_j$ since by definition $\sum_{y \in Y} \eta(\alpha)\pi_0(y) = \sum_{j \in \hat{J}} x_j$. Hence, $\pi(\alpha)$ cannot strictly exceed the payoff from selling assets separately, as defined in (4).

Proof of Proposition 4

The condition on w_k ensures that whatever the bundling, the price of α is the maximum of the draws of the various investors. If α and X are separate, the issuer gets $|\alpha| \, E[\max_k \widetilde{X}_k(\alpha)] + X$. If α and X are bundled then the issuer gets $(|\alpha| + 1)E[\max_k \widetilde{X}_k(\alpha \cup X)]$. Note that $|\alpha| \, E[\max_k \widetilde{X}_k(\alpha)] + X$ is the same as $(|\alpha| + 1)E[\widetilde{\widetilde{X}}(\alpha)]$ where $\widetilde{\widetilde{X}}(\alpha) = \max_k[\frac{|\alpha|}{|\alpha|+1}\widetilde{X}_k(\alpha) + \frac{1}{|\alpha|+1}X]$ where $+$ denotes here the classic addition. When α and X are bundled, $\widetilde{X}_k(\alpha \cup X)$ is the lottery $\frac{|\alpha|}{|\alpha|+1}\widetilde{X}_k(\alpha) \oplus \frac{1}{|\alpha|+1}X$. Thus, it is a mean preserving spread of $\frac{|\alpha|}{|\alpha|+1}\widetilde{X}_k(\alpha) + \frac{1}{|\alpha|+1}X$. Since for any three independent random variables, $\widetilde{Y}_1, \widetilde{Y}_1', \widetilde{Y}_2$ such that $\widetilde{Y}_1'$ is a mean preserving spread of $\widetilde{Y}_1$, we have that $E[\max(\widetilde{Y}_1', \widetilde{Y}_2)] > E[\max(\widetilde{Y}_1, \widetilde{Y}_2)]$ (this can be verified noting that $y_1 \to E_{y_2}[\max(y_1, y2)]$ is a convex function of y_1), we can conclude that $(|\alpha| + 1)E[\max_k \widetilde{X}_k(\alpha \cup X)] > |\alpha| \, E[\max_k \widetilde{X}_k(\alpha)] + X$ and thus full bundling is optimal.

Proof of Proposition 5

The payoff from offering the full bundle $\{X_1, X_2, X_3\}$ is

$$\begin{cases} \min(3x_3, W/3) & \text{for} \quad W \geq 9x_2 \\ \min(3x_2, 2W/3) & \text{for} \quad W \in [9x_2/2, 9x_1/2) \\ \min(3x_1, W) & \text{for} \quad W < 9x_1/2 \end{cases}$$

Suppose instead the bank offers the bundle $\{X_1, X_2\}$ and $\{X_3\}$ as separate asset.

We show that the payoff for the bank is

$$\begin{cases} \min(W, 2x_2 + x_3) & \text{for} \quad 2x_2 < x_3 \\ \min(W/2, 2x_2) + \min(W/2, x_3) & \text{for} \quad 2x_2 > x_3. \end{cases} \tag{15}$$

In these computations we never consider the possibility that the price of the bundle is driven by its lowest evaluation, since in that case it is clear that bundling cannot strictly dominate full separation. Consider first a candidate equilibrium in which investors who sample x_2 from the (x_2, x_1) bundle are indifferent between trading the single asset x_3 and the bundle. That requires $2x_2/p_2 = x_3/p_3$, where p_2 is the price of the bundle and p_3 is the price of the asset x_3. In addition, we need that $p_2 + p_3 \leq W$, so aggregate wealth is enough to buy prices p_2 and p_3. The above conditions give $p_2 \leq 2Wx_2/(x_3 + 2x_2)$ and $p_3 \leq Wx_3/(x_3 + 2x_2)$. In addition, we need that $p_2 \leq W/2$,so those investors who have valuation x_2 for the (x_2, x_l) bundle can indeed drive the price to p_2. Suppose $\frac{2x_2}{x_3+2x_2} < \frac{1}{2}$ that is $2x_2 < x_3$. Then we must have $p_2 = 2Wx_2/(x_3+2x_2)$, and $p_3 = Wx_3/(x_3+2x_2)$.So the payoff of the bank is $\min(W, 2x_2 + x_3)$ when $2x_2 < x_3$. Suppose $2x_2 > x_3$. Then we must have $p_2 = W/2$, and $p_3 = x_3W/4x_2$. That cannot be in equilibrium since investors who sample x_l still have money and would like to drive the price p_3 up. So if $2x_2 > x_3$ investors are indifferent only if $p_2 = 2x_2$ and $p_3 = x_3$. That requires $W > 4x_2$. If $W < 4x_2$, then we must have $p_2 < 2x_2\frac{p_3}{x_3}$. If $W \in (2x_3, 4x_2)$, we have $p_2 = \frac{W}{2}$ and $p_3 = x_3$. If $W < 2x_3$, we have $p_2 = p_3 = \frac{W}{2}$.

Consider the other possible partitions. Since $2x_3 > x_j$ for $j = 1, 2$, the payoff follows the second case on the payoff in (15). Hence, for each $j = 1, 2$, keeping X_j and offering the bundle $\{X_l, X_3\}$ where $x_j \neq x_l$ gives payoff $\min(W/2, 2x_3) + \min(W/2, x_j)$. Comparing the various payoffs, one can see that when $W < 2(x_1 + x_2)$, no bundling can strictly dominate full separation. For $W > 2(x_1 + x_2)$, the bundling $\{X_1, X_2\}$ and $\{X_3\}$ is optimal until $x_3 + 2x_2 = x_2 + W/2$, that is for $W \in [2x_1 + 2x_2, 2x_3 + 2x_2)$, the bundling $\{X_1, X_3\}$ and $\{X_2\}$ is optimal until $3x_3 + x_2 = W/3$. For $W/3 > 3x_3 + x_2$, the full bundle is optimal. Notice also that the bundling $\{X_2, X_3\}$ and $\{X_1\}$ is dominated by $\{X_1, X_3\}$ and $\{X_2\}$ for $W > 2(x_1 + x_2)$.

Proof of Proposition 6

For $J \leq 3$, see Proposition 5. Suppose $J > 3$ and $\tau = 0$. If $W > \max(2(x_2 + x_1), \sum_j x_j)$, full separation gives $\sum_j x_j$. Suppose the bank bundles assets $\{X_1, X_2\}$ and sells the other assets separately. Consider first a candidate equilibrium in which investors who sample x_2 from the bundle are indifferent between trading the single asset x_j and the bundle. That requires $2x_2/p_2 = x_j/p_j$ for all $j > 2$, where p_2 is the price of the bundle and p_j is the price of the asset x_j. In addition, we need that $p_2 + \sum_{j>2} p_j \leq W$, so aggregate wealth is enough to buy prices p_2 and p_j. The above conditions give $p_2 \leq 2x_2W/(\sum_{j>2} x_j + 2x_2)$, and $p_j \leq x_jW/(\sum_{j>2} x_j + 2x_2)$. In addition, we need that $p_2 \leq W/2$ so those investors who have valuation x_2 for the (x_2, x_1) bundle can indeed drive the price to p_2. Since $2x_2 < \sum_{j>2} x_j$ for $J > 3$, we have $\frac{2x_2}{\sum_{j>2} x_j + 2x_2} < \frac{W}{2}$ and so $p_2 = \min(2x_2, \frac{2x_2}{\sum_{j>2} x_j + 2x_2}W)$ and $p_j = \min(x_j, \frac{x_j}{\sum_{j>2} x_j + 2x_2}W)$ for $j > 2$.So the payoff of the bank is $\min(W, 2x_2 + \sum_{j>2} x_j)$ which exceeds $\sum_j x_j$. Suppose $W \leq \max(2(x_2 + x_1), \sum_j x_j)$. If $W \leq \sum_j x_j$, then no bundling strictly dominates full separation due to Proposition 2. If $W \in (\sum_j x_j, 2(x_2 + x_1)]$, we must have $\sum_j x_j < 2(x_2 + x_1)$, that contradicts the requirement $J > 3$.

Proof of Proposition 7

The payoff from the single bundle is $\pi^1 = \min(2(\chi_1 + \chi_2)z, \frac{\chi_2}{\chi_1+\chi_2}W)$. The payoff from the two identical bundles is $\pi^2 = \min(2(\chi_1 + \chi_2)z, (1 - (\frac{\chi_1}{\chi_1+\chi_2})^2)W)$. Notice that $\pi^2 \geq \pi^1$ since $1 - (\frac{\chi_1}{\chi_1+\chi_2})^2 > 1 - (\frac{\chi_1}{\chi_1+\chi_2}) = \frac{\chi_2}{\chi_1+\chi_2}$.

Proof of Proposition 8

We first show that payoffs in case of deviation π^{-B} are as defined in (7) for $N > 3$ and as $\min((\frac{1}{2})^{N-1}W, x_2)$ for $N \leq 3$. Consider first a candidate equilibrium in which investors who sample x_2 from at least one (x_2, x_1) bundle are indifferent between trading the single asset x_2 and any of those bundles. That requires

$$p^B = 2p^j, \tag{16}$$

where p^B is the price of the bundle and p^j is the price of the asset x_2. In addition, we need that

$$(N-1)p^B + p^j \leq W, \tag{17}$$

so aggregate wealth is enough to buy all bundles at prices p^B and p^j. Conditions (16) and (17) give $p^j \leq W/(2N-1)$, and $p^B \leq 2W/(2N-1)$. In addition, we need that

$$(N-1)p^B \leq (1 - (\frac{1}{2})^{N-1})W, \tag{18}$$

so those investors who have valuation x_2 for at least one of the (x_2, x_1) bundles can indeed drive the price to p^B. Notice that for $N \leq 3$, condition (18) is violated when condition (17) binds so we must have $p^B \leq \min(2x_2, \frac{1}{N-1}(1 - (\frac{1}{2})^{N-1})W)$. This implies that in order to meet condition (16) we need $p^j \leq \min(x_2, \frac{1}{2(N-1)}(1 -$

$(\frac{1}{2})^{N-1})W)$. Suppose that $W < 2(N-1)x_2/(1-(\frac{1}{2})^{N-1})$,and so

$$p^j \leq \frac{1}{2(N-1)}(1-(\frac{1}{2})^{N-1})W. \tag{19}$$

Notice that at any price satisfying condition (19) investors perceive gains from trade from the x_2 asset ($p^j < x_2$) and they still have some wealth to buy it. That is not possible in equilibrium. It follows that when $N \leq 3$ condition (16) can only be satisfied if $p^B = 2x_2$ and $p^j = x_2$, that is when $W \geq 2(N-1)x_2/(1-(\frac{1}{2})^{N-1})$. For $N \geq 4$, instead we can sustain condition (16) for $p^B = 2W/(2N-1)$ and

$$p^j = \frac{W}{2N-1}, \tag{20}$$

Consider then a candidate equilibrium in which $p^B > 2p^j$.That implies that no investors would be willing to buy the bundle, which contradicts $p^B > 2p^j$. Hence, it is not possible to have $p^B > 2p^j$. Finally, consider a candidate equilibrium in which $p^B < 2p^j$, so that the only potential buyers of the deviating bank are those investors who sample x_1 from all bundles. That would give $p^B = \min(2x_2, \frac{1}{N-1}(1-(\frac{1}{2})^{N-1})W)$, and

$$p^j = \min(x_2, (\frac{1}{2})^{N-1}W). \tag{21}$$

This is consistent with $p^B < 2p^j$ only if $N \leq 3$. Collecting the expressions (20) and (21) and noticing that $(\frac{1}{2})^{N-1} > 1/(2N-1)$ iff $N \leq 3$ we have π^{-B} as defined in (7). To see that bundling is an equilibrium for all W when $N \geq 3$, notice that $\frac{1}{N}(1-(\frac{1}{2})^N) \geq \frac{1}{2N-1}$ for $N \geq 2$ and $\frac{1}{N}(1-(\frac{1}{2})^N) > (\frac{1}{2})^{N-1}$ for $N = 3$.

Proof of Proposition 9

We first show that payoffs in case of deviation π^{-U} are as defined in (9). Investors who sample x_2 from the (x_2, x_1) bundle are indifferent between the bundle and one of the x_2 assets if

$$p_2 = 2p_1, \tag{22}$$

where p_2 is the price of the bundle and p_1 is the price of the single x_2 asset. Together with condition $(N-1)p_1 + p_2 = W$, we have

$$p_2 = \frac{2}{N+1}W. \tag{23}$$

Notice that $p_2 < 2p_1$ cannot be sustained in equilibrium since that would give $p_2 = \min(2x_2, \frac{1}{2}W)$,and $p_1 = \min(x_2, \frac{1}{2(N-1)}W)$, which is not consistent with $p_2 < 2p_1$ for $N \geq 3$. Notice also that if $p_2 > 2p_1$, no investor would be willing to buy the bundle thereby contradicting $p_2 > 2p_1$. To show the proposition, notice that we have $\pi^U \geq \pi^{-B}$ since $\frac{1}{2N-1} < \frac{1}{N}$ for all N and $\pi^{-U} \geq \pi^B$ since $\frac{1}{N}(1-(\frac{1}{2})^N) < \frac{2}{N+1}$ for all N .

Proof of Lemma 1

Define $\Delta = \pi^B - \pi^{-B} - (\pi^{-U} - \pi^U)$. We show that $\Delta \leq 0$ for all W. We show this for $N \geq 4$ (the case $N \leq 3$ follows the same logic with slightly different algebra.) From (6) and (7) and given $2N-1 < \frac{2}{\frac{1}{N}(1-(\frac{1}{2})^N)}$, we have

$$\pi^B - \pi^{-B} = \begin{cases} \frac{1}{N}(1-(\frac{1}{2})^N)W - \frac{1}{2N-1}W & \text{for} \quad W < (2N-1)x_2 \\ \frac{1}{N}(1-(\frac{1}{2})^N)W - x_2 & \text{for} \quad W \in [(2N-1)x_2, \frac{2x_2}{\frac{1}{N}(1-(\frac{1}{2})^N)}) \\ x_2 & \text{for} \quad W \geq \frac{2x_2}{\frac{1}{N}(1-(\frac{1}{2})^N)}. \end{cases}$$

From (9) and (8), we have

$$\pi^{-U} - \pi^U = \begin{cases} \frac{2}{N+1}W - \frac{W}{N} & \text{for} \quad W < Nx_2 \\ \frac{2}{N+1}W - x_2 & \text{for} \quad W \in [Nx_2, (N+1)x_2) \\ x_2 & \text{for} \quad W \geq (N+1)x_2. \end{cases}$$

Notice $N < (N+1) < (2N-1) < \frac{2}{\frac{1}{N}(1-(\frac{1}{2})^N)}$, so we have

$$\Delta = \begin{cases} \frac{1}{N}(1-(\frac{1}{2})^N)W - \frac{1}{2N-1}W - (\frac{2}{N+1}W - \frac{W}{N}) & \text{for} \quad W < Nx_2 \\ \frac{1}{N}(1-(\frac{1}{2})^N)W - \frac{1}{2N-1}W - (\frac{2}{N+1}W - x_2) & \text{for} \quad W \in [Nx_2, (N+1)x_2) \\ \frac{1}{N}(1-(\frac{1}{2})^N)W - \frac{1}{2N-1}W - x_2 & \text{for} \quad W \in [(N+1)x_2, (2N-1)x_2) \\ \frac{1}{N}(1-(\frac{1}{2})^N)W - 2x_2 & \text{for} \quad W \in [(2N-1)x_2, \frac{2x_2}{\frac{1}{N}(1-(\frac{1}{2})^N)}) \\ x_2 - x_2 & \text{for} \quad W \geq \frac{2x_2}{\frac{1}{N}(1-(\frac{1}{2})^N)}. \end{cases}$$

It is immediate to see that the expression in the first row is negative for $N > 1$. In the second row, it is enough to show that the expression is negative at $W = Nx_2$. That writes as $\frac{1}{N}(1-(\frac{1}{2})^N)Nx_2 - \frac{1}{2N-1}Nx_2 - (\frac{2}{N+1}Nx_2 - x_2)$, that is negative for $N > 1$. In the third row, it is enough to show that the expression is negative at $W = (2N-1)x_2$. That writes as $\frac{1}{N}(1-(\frac{1}{2})^N)(2N-1)x_2 - \frac{1}{2N-1}(2N-1)x_2 - x_2$, that is negative for $N > 1$. The fourth row is the same as in the case $N < 4$. Hence, $\pi^B - \pi^{-B} - (\pi^{-U} - \pi^U) \leq 0$ for all W.

Proof of Proposition 11

Step 1. We show that full bundling is an equilibrium for $W \geq W_2$ when $N \geq 3$. To see this, we first show that if bank j deviates and withdraws one x_1

asset and offers the bundle (x_2, x_1) its payoff is $\pi^{-F} = x_1 + \min(\pi_0^{-F}, 2x_2)$, where

$$\pi_0^{-F} = \max(\frac{1}{2}(\frac{2}{3})^{N-1}W, \frac{2}{3N-1}(1 - \frac{1}{2}(\frac{2}{3})^{N-1})W).$$

In π_0^{-F}, the first term corresponds to the case in which the only potential buyers of the deviating bank are those investors who sample x_2 from the deviating bank and x_1 from all the other bundles and the second term corresponds to the case in which investors who sample x_2 from the (x_2, x_1) bundle and x_2 from at least one (x_2, x_1, x_1) bundle are indifferent between trading any of those bundles. Following the same logic used to derive π^{-B} in Proposition 8, one can easily show that the first term applies for $N \leq 3$ and the second term for $N > 3$. To see when full bundling is an equilibrium, notice that $\frac{2}{3N-1}(1 - \frac{1}{2}(\frac{2}{3})^{N-1}) < \frac{1}{N}(1 - (\frac{2}{3})^N)$ for all N. Notice also that $\frac{1}{2}(\frac{2}{3})^{N-1} < \frac{1}{N}(1 - (\frac{2}{3})^N)$ for $N = 3$.

Step 2. We show that full bundling is dominated by partial bundling, that is $\pi^F < \pi^P$, when $W < 2W_2$. The payoff π^F is increasing linearly in W up to $W^F = 3Nx_2(1 - (\frac{2}{3})^N)^{-1}$, and it is equal to $3x_2$ afterwards. The payoff π^P is increasing linearly in W up to $W^P = 2Nx_2(1 - (\frac{1}{2})^N)^{-1}$, and it is equal to $2x_2$ afterwards. Since $\pi^F = \pi^P = 0$ at $W = 0$, $W^F > W^P$, and $\frac{d\pi^F}{dW} < \frac{d\pi^P}{dW}$ for $W < W^P$, it follows that $\pi^F < \pi^P$ when $W < 2W_2$.

Step 3. We show that if full bundling is an equilibrium then partial bundling is not an equilibrium. We first show that if all banks offer the bundle (x_2, x_1) and bank j deviates by offering (x_2, x_1, x_1) its payoff is $\pi^{-P} = \min(3x_2, \pi_0^{-P})$, where

$$\pi_0^{-P} = \max(\frac{1}{3}W, \frac{3}{2N+1}(1 - \frac{2}{3}(\frac{1}{2})^{N-1})W). \tag{24}$$

As in π_0^{-F}, the first term corresponds to the case in which investors who sample x_2 from a (x_2, x_1) bundle and x_2 from the (x_2, x_1, x_1) bundle prefer buying the latter while the second term corresponds to the case in which these investors are indifferent between the two. To see that if full bundling is an equilibrium then partial bundling is not an equilibrium, notice that $\frac{3}{2N+1}(1-\frac{2}{3}(\frac{1}{2})^{N-1}) > \frac{1}{N}(1-(\frac{1}{2})^N)$ for all N. Notice that $\frac{1}{3} > \frac{1}{N}(1 - (\frac{1}{2})^N)$ for $N = 3$. Hence, we have $\pi^{-P} > \pi^P$ for all W when $N \geq 3$.

Proof of Proposition 13

To see that bundling is an equilibrium for $W \geq 8x_2/3$ when $N = 2$, notice that the payoffs in case of deviations are as defined in the proof of Proposition 8 and that $\frac{1}{N}(1 - (\frac{1}{2})^N) < (\frac{1}{2})^{N-1}$ for $N = 2$. Hence, we have $\pi^B \geq \pi^{-B}$ iff $\frac{1}{N}(1 - (\frac{1}{2})^N)W \geq x_2$, that is $W \geq 8x_2/3$.

Proof of Proposition 14

Point i). Suppose all banks bundle. Consider an investor drawing x_1 from first bundle and x_2 from the second bundle. When the price p of each asset in each bundle is p, buying is preferred to short selling if

$$p < \frac{x_2}{1+\theta}. \tag{25}$$

Suppose that is the case, the price of each asset in each bundle is given by $\frac{3}{8}\frac{W}{p} = 2 + \frac{1}{8}\frac{\theta w}{p}$, in which the demand for the bundle comes from investors who have sampled x_2 from at least one bundle (equally shared among the two banks). That gives

$$p = \frac{3-\theta}{16}W. \tag{26}$$

Conditions (25) and (26) give $W < 16x_2/(1+\theta)(3-\theta)$.Collecting (25) and (26) we have that the payoff of each bank is $2p = \min(\frac{3-\theta}{8}W, \frac{2x_2}{1+\theta})$ for $W < 16x_2/(1+\theta)(3-\theta)$. Suppose now condition (25) is violated. The price of each asset in each bundle is given by $\frac{1}{8}\frac{W}{p} = 2 + \frac{3}{8}\frac{\theta w}{p}$, in which the demand for the bundle comes from investors who have sampled x_2 from all bundles (equally shared among the two banks). That gives $p = (1-3\theta)W/16$, and so the payoff of each bank is $2p = \min(\frac{1-3\theta}{8}W, 2x_2)$ for $W \geq 16x_2/(1+\theta)(1-3\theta)$. That defines π^θ as

$$\pi^\theta = \begin{cases} \min(\frac{3-\theta}{8}W, \frac{2x_2}{1+\theta}) & \text{for} \quad W \leq \frac{16x_2}{(1+\theta)(1-3\theta)} \\ \min(\frac{1-3\theta}{8}W, 2x_2) & \text{for} \quad W \geq \frac{16x_2}{(1+\theta)(1-3\theta)}. \end{cases} \tag{27}$$

Suppose one bank deviates. If the price of the single asset x_2 is p, an investor drawing x_1 from the bundle prefers buying the single asset x_2 rather then short selling the bundle if condition (25) holds. In that case, both the bundle and the single asset attract half of the aggregate wealth. The payoff of the deviating bank is $\min(\frac{1}{2}W, \frac{x_2}{1+\theta})$. If condition (25) is violated, an investor drawing x_1 from the bundle prefers short selling the bundle rather than buying the single asset x_2. Only those who sample x_2 from the bundle are willing to buy the single asset x_2. If p_1 is the price of the single asset x_2 and p_2 is the price of each asset in the bundle, we need

$$p_1 = p_2. \tag{28}$$

Denote with λ the fraction of investors who draw x_2 from the bundle and buy the bundle. For the bundle, we have $\lambda\frac{W}{2p_2} = 2 + \frac{\theta W}{2p_2}$. For the single asset, we have $(1-\lambda)W = 2p_1$. Together with (28), these conditions give $\lambda = (2+\theta)/3$, and so $p_1 = p_2 = W(1-\theta)/6$. As we need $p > x_2/(1+\theta)$, we need $W > 6x_2/((1-\theta)(1+\theta))$. Hence, the payoff of the deviating bank is $\min(\frac{1-\theta}{6}W, x_2)$ for $W > 6x_2/((1-\theta)(1+\theta))$. That defines $\pi^{-\theta}$ as

$$\pi^{-\theta} = \begin{cases} \min(\frac{1}{2}W, \frac{x_2}{1+\theta}) & \text{for} \quad W \leq \frac{6x_2}{(1-\theta)(1+\theta)} \\ \min(\frac{1-\theta}{6}W, x_2) & \text{for} \quad W \geq \frac{6x_2}{(1-\theta)(1+\theta)}. \end{cases} \tag{29}$$

Finally, notice that $\pi^\theta \geq \pi^{-\theta}$ when $W \geq W^\theta$, where $\frac{3-\theta}{8}W^\theta = \frac{x_2}{1+\theta}$. In fact, the only intersection between π^θ and $\pi^{-\theta}$ is at W^θ.

Point ii). We need to show that if bundling is an equilibrium then no bundling is not an equilibrium. Suppose no bank bundles and one bank deviates and offers the bundle. Its payoff are defined in the proof defining $\pi^{-\theta}$ (since we only have 2 banks) and they write as

$$\pi^{-U} = \begin{cases} \min(\frac{1}{2}W, \frac{2x_2}{1+\theta}) & \text{for} \quad W \leq \frac{6x_2}{(1-\theta)(1+\theta)} \\ \min(\frac{1-\theta}{3}W, 2x_2) & \text{for} \quad W > \frac{6x_2}{(1-\theta)(1+\theta)}. \end{cases}$$

We have $\pi^{-U} > \pi^U$ for $W > 2x_2$. Since $2x_2 < 8x_2/(3-\theta)(1+\theta)$,which is the minimal wealth required to have bundling in equilibrium, we have that if bundling is an equilibrium then no bundling is not an equilibrium.

Proof of Proposition 15

Suppose $N = M = 2$. As there are 4 assets and 2 signals to be drawn, there are 6 possible realizations of the draws, each with equal probability. Suppose all banks bundle. A fraction 1/6 of investors sample two signals from the first bundle, 1/6 of investors sample two signals from the second bundle and the remaining investors sample one signal from each bundle. Hence, for each bundle, 1/3 of investors have valuation $2x_2$, 1/6 of investors have valuation $x_1 + x_2$, 1/3 of investors have valuation $2x_1$ and 1/6 of investors have no valuation (and do not buy that bundle).

Each bundle can be sold at price $2x_2$ when the fraction of investors who draw x_2 from at least one bundle (that is, half of the investors) have enough wealth. That requires $W/2 > 4x_2$. When $W/2 < 2(x_1 + x_2)$, investors with valuation $x_1 + x_2$ start buying and so each bundle attracts a fraction 5/12 of wealth. The payoff for each bank is

$$\pi_2^B = \begin{cases} \min(\frac{1}{4}W, 2x_2) & \text{for} \quad W \geq 4x_2, \\ \min(\frac{5}{12}W, x_2) & \text{for} \quad W < 4x_2. \end{cases}.$$

Suppose bank j deviates and sell the assets separately, denote its payoff as π_2^{-B}. It can be easily shown that in equilibrium investors who sample x_2 from the bundle and a single asset x_2 always prefer to buy the bundle. Hence, bank j attracts only those investors who sample x_2 from j and do not sample x_2 from the bundle (that is, 1/3 of investors). We have $\pi_2^{-B} = \min(W/3, x_2)$. The result follows from noticing that $\pi_2^B \geq \pi_2^{-B}$ for all W.

Proof of Proposition 16

We first show that the payoff from tranching writes as

$$\pi^T = \begin{cases} \min(\frac{W}{2}, 2(x_1 + x_2)) & \text{for} \quad W \geq 2W_4. \\ \min(W, W_4) & \text{for} \quad W < 2W_4. \end{cases} \tag{30}$$

To see this, notice first that investor drawing signal x_z for $z = \{1, 2\}$ prefers to buy the junior tranche as opposed to the senior tranche iff $p_s/p_j \geq s_z/j_z$.Notice also that $s_2/j_2 < s_1/j_1$,since $x_1 < x_2$. This implies that in equilibrium it must be that

$$\frac{p_s}{p_j} = \frac{s_2}{j_2}. \tag{31}$$

Suppose by contradiction that $p_s/p_j < s_2/j_2$. Then everyone would strictly prefer buying the senior tranche and no one would buy the junior tranche, which cannot happen in equilibrium. Suppose instead

$$\frac{p_s}{p_j} > \frac{s_2}{j_2}. \tag{32}$$

Those who sample x_2 only buy the junior tranche while those who sample x_1 must buy the senior tranche. Notice first that we must have $W/2 \leq j_2$, or those who sample x_2 would still have money left and they would be willing to buy the senior tranche given $p_s \leq s_1 < s_2$. Suppose $W/2 \leq s_1$. Then those who sample x_1 only buy the senior tranche and $p_s = W/2$. We would have $p_s = p_j = W/2$ and would contradict (32) since $s_2 > j_2$. Suppose $W/2 > s_1$, which is consistent with $W/2 \leq j_2$ only if $s_1 < j_2$. We would have $p_s = s_1$ and $p_j = W/2$, which contradicts (32) since $s_2 > j_2$ and $W/2 > s_1$. Hence, it cannot be that those who sample x_2 only buy either the junior tranche or the senior tranche.

Hence, for $W > 2W_4$, only those who sample x_2 buy both tranches, in which case $\pi^T = \min(\frac{W}{2}, 2(x_1 + x_2))$. If $W < 2W_4$, investors sampling x_1 are attracted in the market and they only buy the senior tranche. Investors sampling x_2 buy both tranches and we have $p_s = s_1$ and $p_j = \frac{j_2}{s_2} s_1$ due to (31). That gives $\pi^T = s_1(1 + \frac{j_2}{s_2}) = W_4$. That occurs until $W \geq W_4$. If $W < W_4$, we have $\pi^T = W$. The payoff from offering the bundle writes as

$$\pi^B = \begin{cases} \min(\frac{W}{2}, 2(x_1 + x_2)) & \text{for} \quad W \geq 4x_1. \\ \min(W, 2x_1) & \text{for} \quad W < 4x_1. \end{cases}$$

Hence, given (30) and noticing that $W_4 > 2x_1$, we have $\pi^T \geq \pi^B$ for all W and $\pi^T > \pi^B$ for $W \in (2x_1, 2W_4)$.

7.2 Additional Results

7.2.1 Keeping Assets

We revisit some results in the monopolistic setting in the case of $\tau = 1$.

Proposition 17 *Suppose $K \to \infty$ and $J = 3$. If $\tau = 1$, then*

$$\alpha^* = \begin{cases} \alpha_1 = \{X_1, X_2, X_3\} & \textit{for} \quad W \geq 6x_3 + 3x_2 \\ \alpha_0 = X_2,\ \alpha_1 = \{X_1, X_3\} & \textit{for} \quad W \in [2x_3 + 2x_2, 6x_3 + 3x_2) \\ \alpha_0 = X_3,\ \alpha_1 = \{X_1, X_2\} & \textit{for} \quad W \in [2x_1 + 2x_2, 2x_3 + 2x_2) \\ \alpha_0 = \{X_1, X_2, X_3\} & \textit{for} \quad W < 2x_1 + 2x_2. \end{cases}$$

Proof. The payoff from offering the full bundle $\{X_1, X_2, X_3\}$ is unchanged relative to $\tau = 0$. For each $j = 1, 2, 3$, keeping X_j and offering the bundle $\{X_l, X_h\}$ where $x_h > x_l$ gives payoff

$$\begin{cases} x_j + 2x_h & \text{for} \quad W > 4x_h \\ x_j + W/2 & \text{for} \quad W \in [4x_l, 4x_h) \\ x_j + 2x_l & \text{for} \quad W \in [2x_l, 4x_l) \\ x_j + W & \text{for} \quad W < 2x_l. \end{cases}$$

Comparing the various payoffs, one can see that keeping all assets (which gives $\sum x_j$) dominates any other bundle when $x_j + W/2 < \sum x_j$ for all j, that is when $W < 2(x_1 + x_2)$. Keeping asset X_3 is optimal until $x_3 + 2x_2 = x_2 + W/2$, that is for $W \in [2x_1 + 2x_2, 2x_3 + 2x_2)$, then keeping asset X_2 is optimal until $3x_3 + x_2 = W/3$. For $W/3 > 3x_3 + x_2$, the full bundle is optimal. Notice also that keeping asset X_1 is dominated by keeping X_2 for $W > 2(x_1 + x_2)$. ■

Proposition 18 *Suppose $K \to \infty$ and $J > 1$ and $\tau = 1$. Some bundling strictly dominates keeping all assets if and only if $W > 2(x_2 + x_1)$.*

Proof. Suppose $J > 3$ and $\tau = 1$. If $W > 2(x_2 + x_1)$ and the bank bundles assets $\{X_1, X_2\}$ and keeps the other assets, its payoff is $\min(W/2, 2x2) + \sum_{j>2} x_j$, which exceeds the payoff from keeping all assets $\sum_j x_j$. If $W \leq 2(x_2 + x_1)$ any bundle with 2 assets is sold at most at price $W/2$, which is below $x_j + x_{j\prime}$ for each $j, j\prime$. Any bundle with 3 assets α is profitable only if $2W/3$ exceeds $\sum_{j\in\alpha} x_j$ that requires at least $W > 3(x_1 + x_2 + x_3)/2$ and that contradicts $W \leq 2(x_2 + x_1)$. For any bundle $\acute{\alpha}$ with more than 3 assets $W < \sum_{j\in\acute{\alpha}} x_j$ so that cannot be profitable. ■

7.2.2 Discount for Complexity

Consider the effect of discounting for complexity. There are N banks each with two assets with value $x_1 = 0$ and $x_2 > 0$, $\tau = 0$ and a continuum of investors. Suppose investors observe the size of the bundle and when investors sample x_2 from the bundle, their valuation is $2x_2 - \delta$, δ is the discount for complexity.

If $\delta \geq x_2$, there is no incentive to bundle, since a single asset with value x_2 would be perceived as more valuable than a bundle containing one x_2 asset. If $\delta < x_2$, instead, it is immediate to derive that the incentives to bundle for a monopolist do not depend on δ. Consider now an oligopolistic setting. We have:

Proposition 19 *There exists a $\hat{\delta}$ such that if $\delta < \hat{\delta}$ then Propositions 8 and 10 hold. The threshold $\hat{\delta}$ increases in N and $\hat{\delta} \to x_2$ for $N \to \infty$.*

Proof. Suppose all banks bundle and one bank deviates. Following the same logic as in Proposition 8, we can write its payoff as $\pi^{-D} = \min(x_2, \pi_0^{-D})$, where

$$\pi_0^{-D} = \max((\frac{1}{2})^{N-1}W, \frac{x_2}{(2N-1)x_2 - (N-1)\delta}W).$$

Notice that for $N = 3$, $\pi_0^{-D} = W/4$ iff $2\delta < h$. For $N > 3$, $\pi_0^{-D} = \frac{x_2}{(2N-1)x_2-(N-1)\delta}W$. Hence, for $N = 3$ and $2\delta < h$, then bundling is an equilibrium for all W since $\frac{1}{3}(1 - (\frac{1}{2})^3)W > \frac{W}{4}$. When $\pi_0^{-D} = x_2W/((2N-1)x_2 - (N-1)\delta)$, bundling is an equilibrium for all W if

$$\frac{x_2}{(2N-1)x_2 - (N-1)\delta}W < \frac{1}{N}(1 - (\frac{1}{2})^N)W),$$

which defines the upper bound $\hat{\delta}$. If $\delta > \hat{\delta}$, bundling is an equilibrium for $W \geq W_1$. Notice that for $N = 3$, $\delta < \hat{\delta}$ implies $2\delta < h$. Hence, for $N > 2$, bundling is an equilibrium for $W \geq W_1$ when $\delta > \hat{\delta}$ and it is an equilibrium for all W when $\delta \leq \hat{\delta}$.

To show the equivalence with Proposition 10, we need to show that if bundling is an equilibrium then no bundling is not an equilibrium. Suppose all banks offer the assets separately and bank j deviates by offering the bundle (x_2, x_1). Following the same logic as in Proposition 8, we can write its payoff as $\pi^{-U} = \min(2x_2 - \delta, \pi_0^{-U})$, where

$$\pi_0^{-U} = \min(\frac{1}{2}W, \frac{(2x_2 - \delta)}{2x_2 - \delta + (N-1)x_2}W). \tag{33}$$

Notice that for $N \geq 3$, $\pi_0^{-U} = \frac{(2x_2-\delta)}{2x_2-\delta+(N-1)x_2}W$ and $\frac{(2x_2-\delta)}{2x_2-\delta+(N-1)x_2} > \frac{1}{N}$. So for $N \geq 3$ not bundling is never an equilibrium provided that $x_2 > \delta$. ■